Exponential and Logarithmic Functions
Review

Name ______________________________ Date _______

Show all WORK.

1. Give an example of an exponential function that models exponential growth.

$y = ab^x; \quad b > 1 \rightarrow \quad y = \frac{1}{2}(1.5)^x$

2. Give an example of an exponential function that models exponential decay.

$y = ab^x; \quad 0 < b < 1 \quad\quad y = 27(.9)^x$

Write an exponential function of the form $y = ab^x$ that has a graph through the given points.

3. $(2,2),(3,4)$

$2 = ab^2$

$a = \frac{2}{b^2}$

$4 = \frac{2}{b^2} b^3$

$4 = 2b$

$b = 2$

$a = \frac{2}{(2)^2} = \frac{2}{4} = \frac{1}{2}$

$\boxed{y = \frac{1}{2}(2)^x}$

4. Determine the inverse of the function $y = 5^{x+1} - 4$

$x = 5^{y+1} - 4$

$x + 4 = 5^{y+1}$

$\log_5 x + 4 = y + 1$

$\boxed{y = \log_5 x + 3}$

5. You put \$2500 into a bank account earning 4% annual interest. Find the amount after 12 years in the account if the interest is compounded

(a) daily,

$A = 2500\left(1 + \frac{.04}{365}\right)^{12 \cdot 365} = \boxed{\$4040.08}$

(b) compounded continuously.

$A = 2500e^{.04(12)} = \boxed{\$4040.19}$

(c) monthly.

$A = 2500\left(1 + \frac{.04}{12}\right)^{12(12)} = \boxed{\$4036.96}$

(d) What is the actual percentage rate (APR) if it is compounded monthly?

$\left(1 + \frac{.04}{12}\right)^{12} = 1.0407 \rightarrow \boxed{4.07\%}$

6. Describe how the following functions are related to their parent function→how are they translated, stretched, etc.

a. $\log_{.5}(4x-20)$

$= \log_{.5} 4(x-5)$

$= \log_{.5} 4 + \log_{.5}(x-5)$

$= -2 + \log_{.5}(x-5)$

$= \log_{.5}(x-5) - 2$

• PF $= \log_{.5} x$
• Shift right 5, down 2

b. $-2\log(-20x+80)-5$

$= -2\log 10(-x+4)) - 5$

$= -2(\log 10 + \log(-x+4)) - 5$

$= -2(1 + \log(-x+4)) - 5$

$= -2 - 2\log(-x+4) - 5$

$= -2\log(-x+4) - 7$

• PF $= \log x$
• Right 4, down 7
• Vert. stretch of -2
• All input values multiply by -1

Write each equation in logarithmic form.

11. $7^3 = 343$

$\log_7 343 = 3$

12. $\left(\frac{2}{3}\right)^{-3} = \frac{27}{8}$

$\log_{\left(\frac{2}{3}\right)}\left(\frac{27}{8}\right) = -3$

13. $3.2^{x+1} = m$

$\log_{3.2} m = x+1$

Write each equation in exponential form.

14. $\log_3 81 = 4$

$3^4 = 81$

15. $\log_b m = w$

$b^w = m$

16. $\ln 4 = 1.386$

$e^{1.386} = 4$

<u>Without the use of a calculator</u> evaluate each logarithm. SHOW YOUR WORK FOR CREDIT!

17. $\log_5 5 = x$

$5^x = 5$

$x = 1$

18. $\log_{3.2} 1 = x$

$3.2^x = 1$

$x = 0$

19. $\log_2 16 = x$

$2^x = 16$

$2^x = 2^4$

$x = 4$

20. $\log_3 \frac{1}{9} = x$

$3^x = \frac{1}{9}$

$3^x = \frac{1}{3^2}$

$3^x = 3^{-2}$

$x = -2$

21. Solve: round to the nearest thousandth (or leave in exact form if you do at home)

$3^x = 17$

$x \log 3 = \log 17$

$x = \frac{\log 17}{\log 3} \approx 2.579$

22. Solve: round to the nearest thousandth (or leave in exact form if done at home)

$4^{x-5} + 7 = 20$

$4^{x-5} = 13$

$(x-5)\log 4 = \log 13$

$x - 5 = \frac{\log 13}{\log 4}$

$x = \frac{\log 13}{\log 4} + 5 \approx 6.850$

23. Solve: round to the nearest thousandth (or leave in exact form if done at home).

$\log 4x = 2$

$10^2 = 4x$

$100 = 4x$

$X = 25$

24. Solve: round to the nearest thousandth (or leave in exact form if done at home).

$\ln x - 3\ln 2 = 6$

$\ln\left(\frac{x}{2^3}\right) = 6$

$e^6 = \frac{x}{8}$

$8e^6 = x$

$x \approx 3227.430$

Rewrite as a single logarithm, and simplify your answer if possible.

25. $\log_3 3 - 2(\log_3 x - \log_3 y)$

$* \log_3 3 = 1$

$1 - 2\log_3 x + 2\log_3 y = \boxed{1 - \log_3(x^2y^2)}$

26. $5\ln x - \ln x$

$\ln x^5 - \ln x$

$\ln\left(\frac{x^5}{x}\right) = \ln(x^4)$

Expand each logarithm completely.

27. $\ln 4x$

$\ln 4 + \ln x$

28. $\log_7 \frac{x}{6\sqrt[3]{y}}$

$\log_7 x - (\log_7(6\sqrt[3]{y}))$

$\log_7 x - (\log_7 6 + \frac{1}{3}\log_7 y)$

$\log_7 x - \log_7 6 - \frac{1}{3}\log_7 y$

29. Carbon-14 has a half-life of 5730 years. A. Write an exponential decay function for a 24-mg sample. B. How much carbon-14 remains in 30 millennia? (1 millennium = 1000 years). C. How long will it take for there to be 5 mg of the sample remaining?

A) $y = 24\left(\frac{1}{2}\right)^{\frac{t}{5730}}$

B) $y = 24\left(\frac{1}{2}\right)^{\frac{30000}{5730}} = .637$ mg

C) $5 = 24\left(\frac{1}{2}\right)^{\frac{t}{5730}}$

$\frac{5}{24} = \left(\frac{1}{2}\right)^{\frac{t}{5730}}$

$\log\left(\frac{5}{24}\right) = \frac{t}{5730}\log\left(\frac{1}{2}\right)$

$\frac{\log\left(\frac{5}{24}\right)}{\log(1/2)} = \frac{t}{5730}$

$t = \frac{5730\log(5/24)}{\log(1/2)} \approx \boxed{12{,}967.19 \text{ years}}$

30. Graph, List the domain, range and horizontal asymptote.

$f(x) = -(2)^{2x-10} + 2$

$F(x) = -(2)^{2(x-5)} + 2$

$F(x) = -(4)^{x-5} + 2$

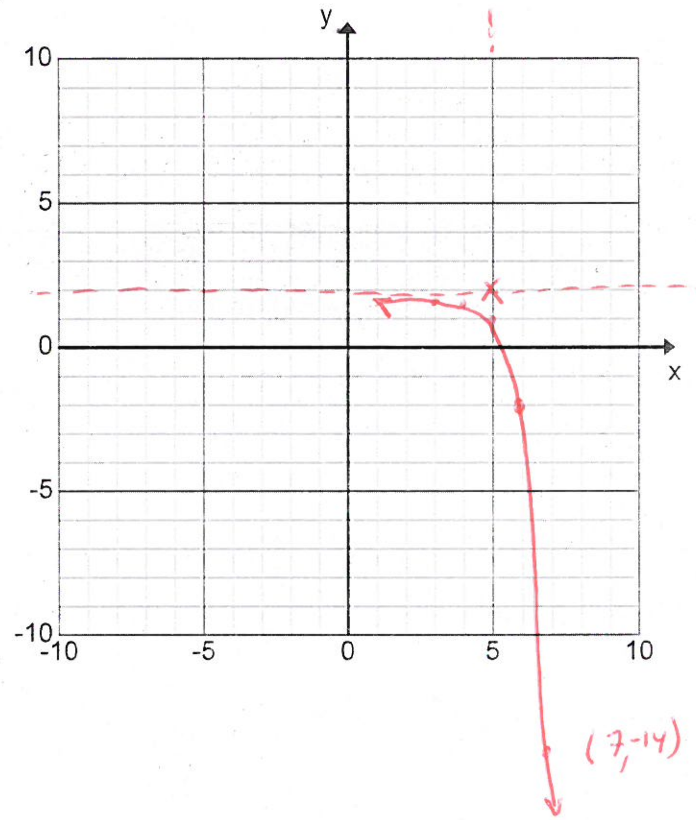
Domain:
Range:
Asymptote:

Domain: $\mathbb{R}$

Range: $y < 2$

Asymptote: $y = 2$

x	4^x
-2	1/16
-1	1/4
0	1
1	4
2	16



31. Graph, list domain, range, and vertical asymptote.

$2\log_3(3x-6)+5$

$= 2\log_3 3(x-2) + 5$

$= 2(\log_3 3 + \log_3 (x-2)) + 5$

$= 2(1 + \log_3(x-2)) + 5$

$= 2 + 2\log_3(x-2) + 5$

$= 2\log_3(x-2) + 7$

Domain: $\mathbb{R}$
Range:
Asymptote:

x	3^x
-2	1/9
-1	1/3
0	1
1	3
2	9

x	$y = 2\log_3 x$
1/9	-4
1/3	-2
1	0
3	2
9	4