6-3 Notes: Solving Systems of Equations by Elimination

Lesson Objective: To solve systems of equations using a *second* algebraic method.

Example: Two groups of students order both burritos and tacos at a local restaurant. One order of 3 burritos and 4 tacos cost \$11.33. The other order of 9 burritos and 5 tacos costs \$23.56. How much does 1 taco and 1 burrito cost?

Define your variables: x = cost of 1 burrito
y = " " " taco

Write a system of equations:

1st order → $3x + 4y = 11.33$

2nd order → $9x + 5y = 23.56$

Why is the substitution method difficult here?

There is no "easy" variable to solve for. None of the variables have a coefficient of 1.

This Concept:

$$\begin{array}{r} 3 + 7 = 10 \\ + \quad 2 + 6 = 8 \\ \hline 5 + 13 = 18 \end{array}$$

Use this idea to solve the systems below:

Combine the two equations to eliminate one variable.

Ex.

$$+\begin{cases} 5x - \cancel{6y} = -32 \\ 3x + \cancel{6y} = 48 \end{cases}$$

$8x = 16$ ← Solve for x

$\frac{8x}{8} = \frac{16}{8}$

$x = 2$

$3(2) + 6y = 48$

$6 + 6y = 48$
$-6 \qquad -6$

$\frac{6y}{6} = \frac{42}{6}$

$y = 7$

(2, 7)

Ex.

$$+\begin{cases} \cancel{7x} + 2y = 10 \\ \cancel{-7x} + y = -16 \end{cases}$$

$\frac{3y}{3} = \frac{-6}{3}$

$y = -2$

$7x + 2(-2) = 10$

$7x - 4 = 10$
$+4 \qquad +4$

$\frac{7x}{7} = \frac{14}{7}$

$x = 2$

(2, -2)

Multiplying one equation by a constant:

a #

Ex.

$$-5\left(2x+5y=-22\right)$$
$$\begin{cases} 2x+5y=-22 \\ 10x+3y=22 \end{cases}$$

$$+\quad -\cancel{10x}-25y=110$$
$$\cancel{10x}+3y=22$$
$$\frac{-\cancel{22}y}{-\cancel{22}}=\frac{132}{-22}$$
$$y=-6$$
$$2x+5(-6)=-22$$
$$2x-30=-22$$
$$+30 \quad +30$$
$$\frac{\cancel{2}x}{\cancel{2}}=\frac{8}{2}$$
$$x=4$$

(4, −6)

Ex.

$$\begin{cases} -2x+15y=-32 \\ 7x-5y=17 \end{cases}$$
$$3\left(7x-5y=17\right)$$

$$+\quad -2x+\cancel{15y}=-32$$
$$21x-\cancel{15y}=51$$
$$\frac{\cancel{19}x}{\cancel{19}}=\frac{19}{19}$$
$$x=1$$
$$7(1)-5y=17$$
$$\cancel{7}-5y=17$$
$$-\cancel{7} \quad -7$$
$$\frac{-\cancel{5}y}{-\cancel{5}}=\frac{10}{-5}$$
$$y=-2$$

(1, −2)

Ex.

Two groups of students order both burritos and tacos at a local restaurant. One order of 3 burritos and 4 tacos cost \$11.33. The other order of 9 burritos and 5 tacos costs \$23.56. How much does 1 taco and 1 burrito cost?

Use Elimination:

HW

Multiplying both equations by a constant: a

Ex.

$$\begin{aligned} 3(&4x+2y=14) \\ 2(&7x-3y=-8) \end{aligned} \rightarrow \begin{aligned} 12x + \cancel{6y} &= 42 \\ 14x - \cancel{6y} &= -16 \end{aligned}$$

$$\frac{26x}{26} = \frac{26}{26}$$

$$x = 1$$

$$4(1) + 2y = 14$$

$$\begin{aligned} 4 + 2y &= 14 \\ -4 \quad & \quad -4 \end{aligned}$$

$$\frac{2y}{2} = \frac{10}{2}$$

$$y = 5$$

$$(1, 5)$$

Ex.

$$\begin{aligned} -5(&3x+5y=10) \\ 3(&5x+7y=10) \end{aligned} \rightarrow \begin{aligned} -\cancel{15x} - 25y &= -50 \\ + \quad \cancel{15x} + 21y &= 30 \end{aligned}$$

$$\frac{-4y}{-4} = \frac{-20}{-4}$$

$$y = 5$$

$$3x + 5(5) = 10$$

$$\begin{aligned} 3x + 25 &= 10 \\ -25 \quad & -25 \end{aligned}$$

$$\frac{3x}{3} = \frac{-15}{3}$$

$$x = -5$$

$$(-5, 5)$$

Ex.

$$\begin{aligned} -3(&2x-5y=17) \\ &6x-15y=51 \end{aligned} \rightarrow \begin{aligned} -\cancel{6x} + \cancel{15y} &= -\cancel{51} \\ \cancel{6x} - \cancel{15y} &= \cancel{51} \end{aligned}$$

$$0 = 0$$

True

Since variables cancelled, and it's a true statement, there are infinite solutions. (same line)

Ex.

$$\begin{aligned} 5(&4x-8y=15) \\ 4(&-5x+10y=-30) \end{aligned}$$

$$\begin{aligned} \rightarrow \cancel{20x} - \cancel{40y} &= 75 \\ + \quad -\cancel{20x} + \cancel{40y} &= -120 \end{aligned}$$

$$0 = -45$$

False

No solutions

Work: Old book Page 391

Solve by elimination

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9. $3x - 10y = -25$
$4x + 40y = 20$

11. $x - 8y = 18$
$-16x + 16y = -8$

13. $88x - 5y = 39$
$-8x + 3y = -1$

27. $5y = x$
$2x - 3y = 7$

17. $3x + 2y = -9$
$-10x + 5y = -5$

21. $10x + 8y = 2$
$8x + 6y = 1$

15. Sales A photo studio that takes school pictures offers several different packages. Let w equal the cost of a wallet-sized portrait, and let ℓ equal the cost of an 8×10 portrait.

a. Write a system of equations that relates the cost of wallet-sized portraits and 8×10 portraits to the cost of the basic and deluxe packages.
b. Find the cost of each type of portrait.