9-6 Notes: Solving Equations Using the Quadratic Formula

Lesson Objective: To solve quadratic equations using a new method.

WARM-UP

Rewrite each quadratic in vertex form by completing the square.

$y = x^2 + 6x - 7$

$y = 2x^2 - 16x - 37$

Solve the following quadratic by completing the square. Find exact solutions. Do not round.

$2x^2 - 5x + 3 = 0$

$$\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$ memorize

DERIVATION OF QUADRATIC FORMULA

solve ax^2+bx+c by completing the square.

$ax^2 + bx + c = 0$

$-c \quad -c$

$$\frac{\cancel{a}x^2}{\cancel{a}} + \frac{bx}{a} = \frac{-c}{a}$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \longrightarrow \left(\frac{b}{2}\right)^2 = \left(\frac{\frac{b}{a}}{2}\right)^2 = \left(\frac{b}{a}\cdot\frac{1}{2}\right)^2 = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{4a}{4a}\left(\frac{c}{a}\right) + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac}{4a^2} + \frac{b^2}{4a^2}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2-4ac}}{2a} - \frac{b}{2a}$$

$-b/2a$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Use the quadratic formula to solve the following quadratic equations.

$2x^2 + 5x + 3 = 0$ (Find exact solutions)

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(3)}}{2(2)}$

$x = \frac{-5 \pm \sqrt{25 - 24}}{4}$

$x = \frac{-5 \pm \sqrt{1}}{4}$

$x = \frac{-5 \pm 1}{4}$

$x = \frac{-5 + 1}{4}$ $x = \frac{-5 - 1}{4}$

$x = \frac{-4}{4}$ $x = \frac{-6}{4}$

$x = -1$ $x = \frac{-3}{2}$

$2x^2 + 4x - 7 = 0$ (Find exact and rounded solutions)

$x = \frac{-4 \pm \sqrt{4^2 - 4(2)(-7)}}{2(2)}$

$x = \frac{-4 \pm \sqrt{16 + 56}}{4}$

$x = \frac{-4 \pm \sqrt{72}}{4}$

Exact

$x = \frac{-4 \pm 8.49}{4}$

$x = \frac{-4 + 8.49}{4}$ $x = \frac{-4 - 8.49}{4}$

$x = \frac{4.49}{4}$ $x = \frac{-12.49}{4}$

$x = 1.1225$ $x = -3.1225$

FINDING THE DISCRIMINANT

From the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant: Part under the square root.

Complete the following table

Equation	$b^2 - 4ac$	Zeroes/Roots
$y = x^2 - 6x + 3$	$(-6)^2 - 4(1)(3) = 36 - 12 = 24$	Two
$y = x^2 - 6x + 9$	$(-6)^2 - 4(1)(9) = 36 - 36 = 0$	One
$y = x^2 - 6x + 12$	$(-6) - 4(1)(12) = 36 - 48 = -12$	no real solutions

Property of the Discriminant

For the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, you can use the discriminant $b^2 - 4ac$ to determine the number of solutions or roots.

Value of $b^2 - 4ac$	Number of Roots or Solutions
Positive	Two
Zero	One
Negative	No real solutions

CHOOSING AN APPROPRIATE METHOD TO SOLVE A QUADRATIC

Method	When to Use
Graphing	Use if you have a graphing calculator handy.
Square Roots	Use if the equation has no x term. → "b-term"
Factoring	Use if you can factor the equation easily.
Completing the Square	Use if the x^2 term is 1, but you cannot factor the equation easily. (Can be used for any quadratic).
Quadratic Formula	Use if the equation cannot be factored easily or at all. (any quadratic equation).

Which method(s) would you choose to solve each equation? Justify your reasoning.

a. $2x^2 - 6 = 0$ — Square roots; there is no x term. (no "b" term).

b. $6x^2 + 13x - 17 = 0$ — Quadratic formula; the equation cannot be factored easily.

c. $x^2 + 2x - 15 = 0$ — Factoring; the equation is easily factorable.

d. $16x^2 - 96x + 45 = 0$ — Quadratic formula; the equation cannot be factored easily, and the numbers are large.

e. $x^2 - 7x + 4 = 0$ — Quadratic formula, completing the square, or graphing; the coefficient of the x^2 term is 1, but the equation is not factorable.

Applied Example:

In a volleyball game, a player on one team spikes the ball over the net when the ball is 10 feet above the court. The spike drives the ball downward with a velocity of 55 feet per second. Players on the opposing team must hit the ball back over the net before the ball touches the floor. How much time does the opposing team have to hit the ball? (How much time before ball hits Floor).

↳ $h(t) = 0$

$$\boxed{h(t) = -16t^2 + v_0 t + h_0}$$

$$h(t) = -16t^2 - 55t + 10$$

$$0 = -16t^2 - 55t + 10$$

$$t = \frac{55 \pm \sqrt{(-55)^2 - 4(-16)(10)}}{2(-16)}$$

Finish

Applied Example:

Projectile Motion You hit a golf ball into the air from a height of 1 in. above the ground with an initial vertical velocity of 85 ft/s. The function $h = -16t^2 + 85t + \frac{1}{12}$ models the height, in feet, of the ball at time t, in seconds. Will the ball reach a height of 115 ft?