Final Exam Review
Algebra I-3 Honors

Sean

COMPETENCY 1: QUADRATICS

1. Write the equation of the following quadratic in the most appropriate form.

$y = a(x-2)^2 + 4$

$1 = a(0-2)^2 + 4$

$1 = a(-2)^2 + 4$

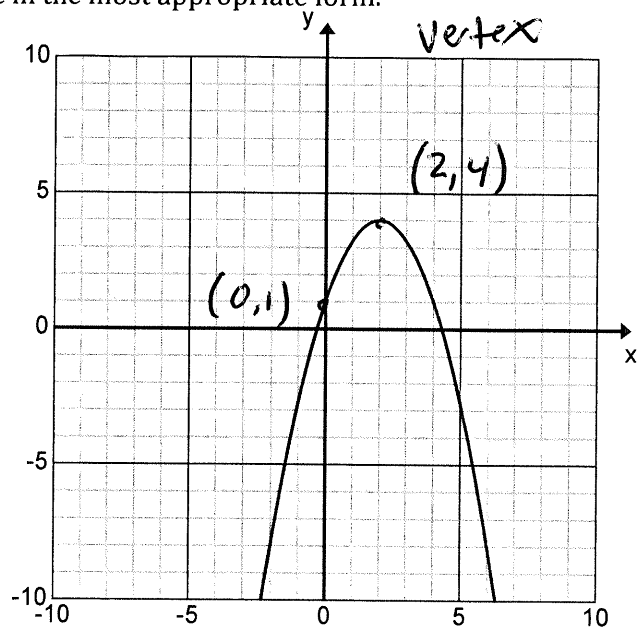
$1 = 4a + 4$

$-4 \qquad -4$

$\frac{-3}{4} = \frac{\cancel{4}a}{\cancel{4}}$

$-\frac{3}{4} = a$

$y = -\frac{3}{4}(x-2)^2 + 4$



2. Write the equation of the following parabola in all forms.

Factored: $y = a(x+2)(x-4)$

$-6 = a(0+2)(0-4)$

$-6 = a(2)(-4)$

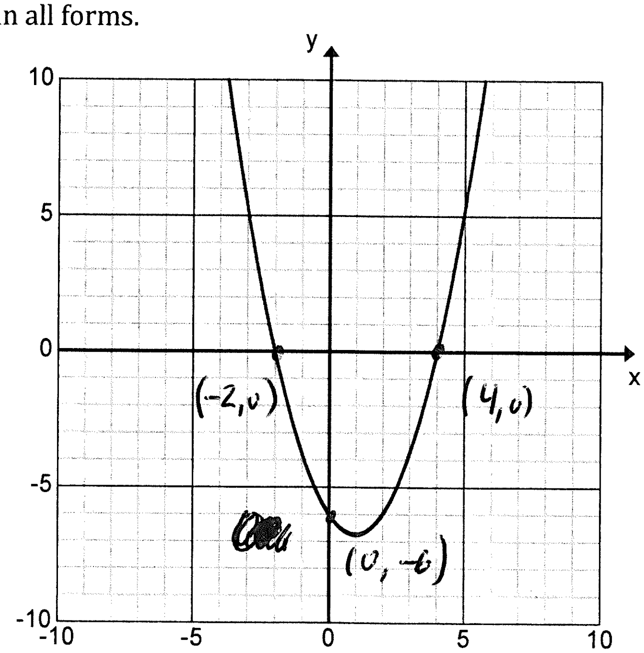
$\frac{-6}{-8} = \frac{a(\cancel{-8})}{\cancel{-8}}$

$\frac{3}{4} = a$

$\boxed{y = \frac{3}{4}(x+2)(x-4)}$

Standard: $y = \frac{3}{4}(x^2 - 2x - 8)$

$\boxed{y = \frac{3}{4}x^2 - \frac{3}{2}x - 6}$



Vertex: axis of sym $\rightarrow x = 1$ (half-way between $x=-2$ and $x=4$)

$y = \frac{3}{4}(1+2)(1-4)$

$y = 3/4(3)(-3)$

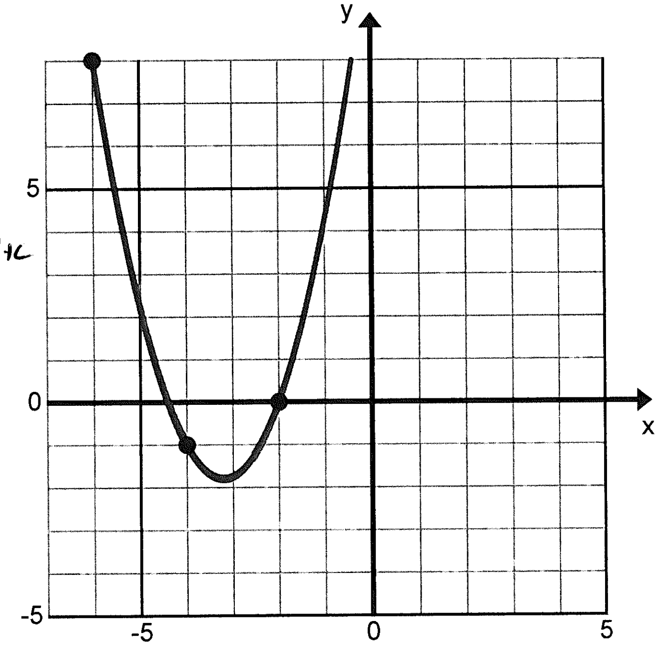
$y = \frac{3}{4}(-9)$

$y = \frac{-27}{4}$

$\boxed{y = \frac{3}{4}(x-1)^2 - \frac{27}{4}}$

$y = ax^2 + bx + c$

3. Write the equation of the parabola shown on the graph in standard form.



Points: $(-6, 8)$, $(-4, -1)$, $(-2, 0)$

① $8 = a(-6)^2 + b(-6) + c$

$8 = 36a - 6b + c$

② $-1 = a(-4)^2 + b(-4) + c$

$-1 = 16a - 4b + c$

③ $0 = a(-2)^2 + b(-2) + c$

$0 = 4a - 2b + c$

④ $-1(0 = 4a - 2b + c) \rightarrow 0 = -4a + 2b - c$

⑤
$$\begin{aligned} 8 &= 36a - 6b + c \\ + \quad 0 &= -4a + 2b - c \\ \hline 8 &= 32a - 4b \end{aligned}$$

$\frac{8}{4} = \frac{32a}{4} - \frac{4b}{4}$

$2 = 8a - b$

⑥
$$\begin{aligned} -1 &= 16a - 4b + c \\ + \quad 0 &= -4a + 2b - c \\ \hline -1 &= 12a - 2b \end{aligned}$$

⑦ $-2(2 = 8a - b)$

$$\begin{aligned} -4 &= 16a + 2b \\ + \quad -1 &= 12a - 2b \\ \hline -5 &= -4a \end{aligned}$$

$\frac{-5}{-4} = \frac{-4a}{-4}$

$\frac{5}{4} = a$

⑧ $2 = 8(5/4) - b$

$2 = 10 - b$

$-10 \quad -10$

$-8 = -b$

$\frac{-8}{-1} = \frac{-b}{-1}$

$8 = b$

⑨ $0 = 4(5/4) - 2(8) + c$

$0 = 5 - 16 + c$

$0 = -11 + c$

$11 = c$

$y = \frac{5}{4}x^2 + 8x + 11$

4. What is the discriminant? What does it tell you?

b^2-4ac

- tells you how many solution a quadratic has:

(1) $b^2-4ac > 0 \rightarrow$ 2 solutions

(2) $b^2-4ac = 0 \rightarrow$ 1 solution

(3) $b^2-4ac < 0 \rightarrow$ 0 real solutions

5. Solve the following quadratic equations by using the QUADRATIC FORMULA. SHOW ALL WORK. Round your solution to the nearest hundredth.

$3.6x^2 - 18x = 2.7$

$3.6x^2 - 18x - 2.7 = 0$

$$x = \frac{18 \pm \sqrt{(-18)^2 - 4(3.6)(-2.7)}}{2(3.6)}$$

$$x = \frac{18 \pm \sqrt{324 + 38.88}}{7.2}$$

$$x = \frac{18 \pm \sqrt{362.88}}{7.2}$$

$$x = \frac{18 \pm 19.05}{7.2}$$

$$\frac{18 + 19.05}{7.2} = \frac{37.05}{7.2} = 5.15$$

$$\frac{18 - 19.05}{7.2} = -.15$$

6. Solve the following quadratic equations by completing the square. Use exact solutions. DO NOT ROUND!

$-2x^2 + 4x - 7 = 0$

$\overline{-2} \quad \overline{-2} \quad \overline{-2} \quad \overline{-2}$

$* \left(\frac{b}{2}\right)^2 = \left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$

$x^2 - 2x + \frac{7}{2} = 0$

$x^2 - 2x + 1 = -\frac{7}{2} + 1$

$x^2 - 2x + 1 = -\frac{7}{2} + \frac{2}{2}$

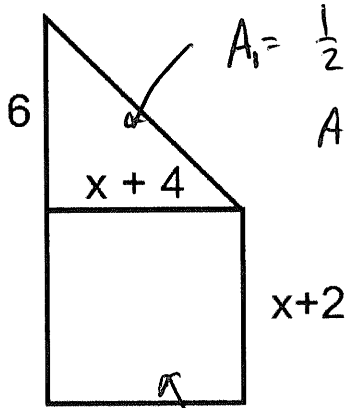
$\sqrt{(x-1)^2} = \sqrt{-\frac{5}{2}}$

$x - 1 = \pm i\sqrt{\frac{5}{2}} + 1$

$+1$

$x = \pm i\sqrt{\frac{5}{2}} + 1$

7. The sum of the areas of the triangle and rectangle below are 25 square inches. **Solve for x to the nearest hundredth.** (Remember the area of a triangle is $A = \frac{1}{2}bh$)



$A_1 = \frac{1}{2}(6)(x+4)$

$A = 3(x+4)$

$A = 3x + 12$

$A = (x+4)(x+2)$

$A_2 = x^2 + 6x + 8$

$25 = A_1 + A_2$

$25 = 3x + 12 + x^2 + 6x + 8$

$25 = x^2 + 9x + 20$

$-25 \qquad -25$

$0 = x^2 + 9x - 5$

$x = \dfrac{-9 \pm \sqrt{9^2 - 4(1)(-5)}}{2(1)}$

$x = \dfrac{-9 \pm \sqrt{81 + 20}}{2}$

$x = \dfrac{-9 \pm \sqrt{101}}{2}$

$x = \dfrac{-9 \pm 10.05}{2}$

$x = \dfrac{-9 + 10.05}{2}$ $\qquad$ $x = \dfrac{-9 - 10.05}{2}$

$x = \dfrac{1.05}{2}$ $\qquad$ $x = \dfrac{-19.05}{2}$

$x = .525$ $\qquad$ $x = -9.525$

$x = .53$ $\qquad$ answer cannot be negative here

8. Write a quadratic function in **standard** form with roots:

$3 + 6i \quad and \quad 3 - 6i$

$(x - (3 + 6i))(x - (3 - 6i))$

$(x - 3 - 6i)(x - 3 + 6i)$

$x^2 - 3x + 6ix - 3x + 9 - 18i - 6ix + 18i - 36i^2$

$x^2 - 6x + 9 - 36(-1)$ $\qquad$ $i^2 = \sqrt{-1}^2 = -1$

$x^2 - 6x + 9 + 36$

$x^2 - 6x + 45$

9. Robin Hood launches an arrow from a starting height of 7 feet. This arrow was calculated to have a starting upwards velocity of about 80 feet per second. Use the following vertical motion equation.

$$h(t) = -16t^2 + v_0 t + h_0$$

$h(t) = -16t^2 + 80t + 7$

Round all solutions to the nearest hundredth.

a) How long will it take the arrow to reach its maximum height?

b) What is the maximum height?

c) How long will it take for the arrow to hit the ground?

d) How long will it take for the arrow to be 100 feet off the ground?

SHOW ALL YOUR WORK HERE: $x = -\frac{b}{2a}$

a) Max height Time: $t = \frac{-80}{2(-16)} = 2.5$

2.5 seconds to reach Max height

b) $h(5) = -16(2.5)^2 + 80(2.5) + 7$

$= -16(6.25) + 200 + 7$

$= -100 + 200 + 7$

$= 100 + 7$

$= 107$

Max height = 107 ft

c) $0 = -16t^2 + 80t + 7$

$t = \frac{-80 \pm \sqrt{(80)^2 - 4(-16)(7)}}{2(-16)}$

$t = \frac{-80 \pm \sqrt{6400 + 448}}{-32} = \frac{-80 \pm \sqrt{6848}}{-32}$

$= \frac{-80 \pm 82.75}{-32}$

$\frac{-80 + 82.75}{-32}$ $\qquad$ $\frac{-80 - 82.75}{-32}$

$-.09$ $\qquad$ 5.09 Seconds

d) $100 = -16t^2 + 80t + 7$

$-100 \qquad -100$

$0 = -16t^2 + 80t - 93$

$t = \frac{-80 \pm \sqrt{(80)^2 - 4(-16)(-93)}}{2(-16)} = \frac{-80 \pm \sqrt{6400 - 5952}}{-32}$

$= \frac{-80 \pm \sqrt{448}}{-32} = \frac{-80 \pm 21.17}{-32}$

$\frac{-80 + 21.17}{-32}$ $\qquad$ $\frac{-80 - 21.17}{-32}$

happens twice → 1.84 seconds $\qquad$ 3.16 Seconds

COMPETENCY TWO: RADICALS AND RADICAL EQUATIONS

Simplify the following radical expressions. Rationalize all denominators. DO NOT ROUND SOLUTIONS. Show Work.

1. $\sqrt{112}$

$16 \cdot 7$

$4\sqrt{7}$

2. $\sqrt{\frac{49}{84}} =$

$\frac{7}{\sqrt{84}} \rightarrow \frac{7}{2\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}}$

$21 \cdot 4$

$\frac{7\sqrt{21}}{2(21)}$

$\frac{\sqrt{21}}{2(3)}$

$\frac{\sqrt{21}}{6}$

3. $8\sqrt{32}+3\sqrt{50}$

$16 \cdot 2$ $25 \cdot 2$

$8(4)\sqrt{2} + 3(5)\sqrt{2}$

$32\sqrt{2} + 15\sqrt{2}$

$\boxed{47\sqrt{2}}$

4. $\sqrt{5}\left(\sqrt{10}+3\sqrt{5}\right)$

$\sqrt{50} + 3(5)$

$25 \cdot 2$

$\boxed{5\sqrt{2} + 15}$

5. $4\sqrt{\frac{3}{8}}$

$= 4\,\frac{\sqrt{3}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{4\sqrt{6}}{2(2)}$

$= \sqrt{6}$

6. $\sqrt{48x^4y^5}$

$16 \cdot 3$ $y^4 \cdot y$

$4x^2y^2\sqrt{3y}$

7. $\sqrt{\frac{54y}{8y^3}}$

$9 \cdot 3$

$= \frac{\sqrt{27}}{\sqrt{4y^2}}$

$= \boxed{\frac{3\sqrt{3}}{2y}}$

8. $\frac{5}{\sqrt{11}+\sqrt{5}} \cdot \frac{\sqrt{11}-\sqrt{5}}{\sqrt{11}-\sqrt{5}}$

$= \frac{5\sqrt{11} - 5\sqrt{5}}{11-5}$

$= \boxed{\frac{5\sqrt{11} - 5\sqrt{5}}{6}}$

Simplify the following radical expressions. Rationalize all denominators. DO NOT ROUND SOLUTIONS. Show Work.

9. $3\sqrt{18x^5}\cdot 2\sqrt{12x}$

$9\cdot 2 \quad x^4\cdot x \quad 4\cdot 3$

$3(3)x^2\sqrt{2x}\cdot 2(2)\sqrt{3x}$

$9x^2\sqrt{2x}\cdot 4\sqrt{3x}$

$36x^2\sqrt{6x^2}$

$36x^2\cdot x\sqrt{6}$

$\boxed{36x^3\sqrt{6}}$

10. $\left(\sqrt{7x}+\sqrt{2}\right)\left(2\sqrt{7x}-3\sqrt{2}\right)$

$2(7x)-3\sqrt{14x}+2\sqrt{14x}-3(2)$

$14x-\sqrt{14x}-6$

11. $\dfrac{\sqrt{8x^3y^5z^2}}{\sqrt{2x^6y^2z^3}}$

$=\dfrac{\sqrt{4y^3}}{\sqrt{x^3z}}$

$=\dfrac{2y\sqrt{y}}{x\sqrt{xz}}\cdot\dfrac{\sqrt{xz}}{\sqrt{xz}}$

$=\dfrac{2y\sqrt{xyz}}{x\cdot xz}$

$=\boxed{\dfrac{2y\sqrt{xyz}}{x^2z}}$

12. $\left(\sqrt{2}+3\right)^2 = (\sqrt{2}+3)(\sqrt{2}+3)$

$= 2+3\sqrt{2}+3\sqrt{2}+9$

$= 11+6\sqrt{2}$

* Don't need to do *

Solve the following radical equations. Check for extraneous solutions. NO DECIMALS.

13. $4\sqrt{x}+5\sqrt{x}=27$

$$\frac{9\sqrt{x}}{9}=\frac{27}{9}$$

$$\sqrt{x}^2=3^2$$

$$x=9$$

14. $\left((2x-3)^{\frac{2}{3}}\right)^{3/2}=(16)^{3/2}$

$$2x-3=(\sqrt{16})^3$$

$$2x-3=(\pm 4)^3$$

$$2x-3=\pm 64$$

$$2x-3=64 \qquad 2x-3=-64$$

$$+3 \quad +3 \qquad +3 \quad +3$$

$$2x=67 \qquad \frac{\cancel{2}x}{\cancel{2}}=\frac{-61}{2}$$

$$x=\frac{67}{2} \qquad x=-\frac{61}{2}$$

15. $\sqrt{3x^2-5x-1}^{\,2}=(x+2)^2$

$$3x^2-5x-1=\cancel{x^2}+\cancel{4x}+\cancel{4}$$

$$-x^2 \quad -4x \quad -4 \qquad \cancel{-x^2}\ \cancel{-4x}\ \cancel{-4}$$

$$2x^2-9x-5=0$$

-10, 2x, 2x, 1, -10, -5, -9

$$(\,x-5)(2x+1)$$

$$x=5 \qquad x=-\frac{1}{2}$$

Check: $\sqrt{3(5)^2-5(5)-1}=5+2$

$$\sqrt{3(25)-25-1}=7$$

$$\sqrt{75-25-1}=7$$

$$\sqrt{49}=7$$

$$7=7$$

$$\sqrt{3\left(-\tfrac{1}{2}\right)^2-5\left(-\tfrac{1}{2}\right)-1}=-\tfrac{1}{2}+2$$

$$\sqrt{3(+1/4)+5/2-1}=3/2$$

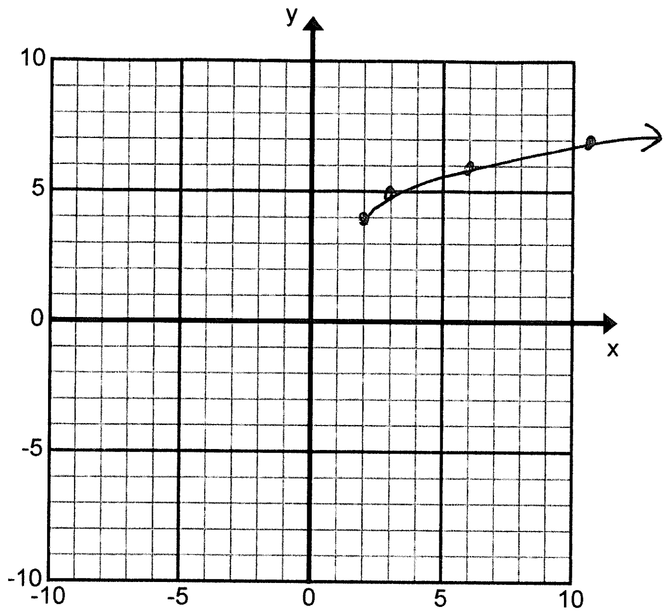
$$\sqrt{3/4+\tfrac{10}{4}-4/4}=3/2$$

$$\sqrt{\tfrac{9}{4}}=3/2$$

$$3/2=3/2$$

Graph the following square root functions. Find the domain and range.

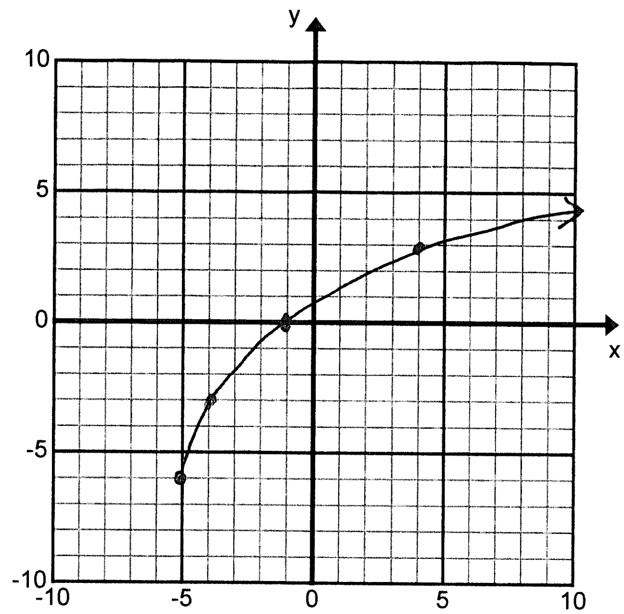
16. $y = \sqrt{x-2} + 4$



Domain: $x \geq 2$

Range: $y \geq 4$

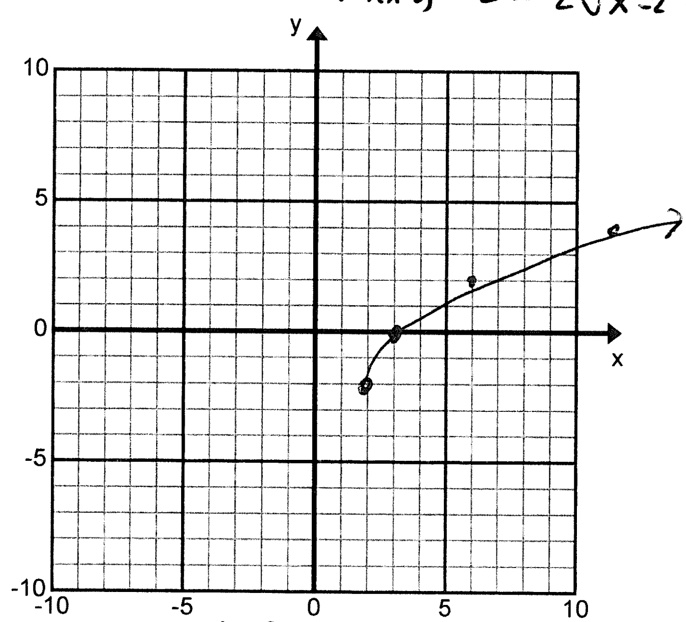
17. $y = 3\sqrt{x+5} - 6$



Domain: $x \geq -5$

Range: $y \geq -6$

18. $y = \sqrt{4x-8} - 2$ $= \sqrt{4(x-2)} - 2 = 2\sqrt{x-2} - 2$



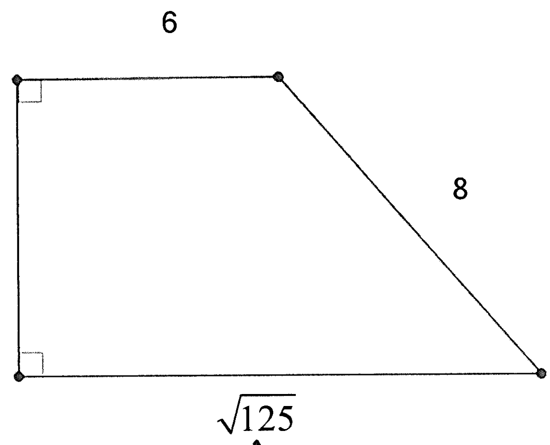
Domain: $x \geq 2$

Range: $y \geq -2$

19. Find the perimeter of the object below. Express your answers in simplified form.

$2\sqrt{5} + 5\sqrt{5} + 6 + 8$

$7\sqrt{5} + 14$



$4 \cdot 5$

$2\sqrt{5}$

$25 \cdot 5$

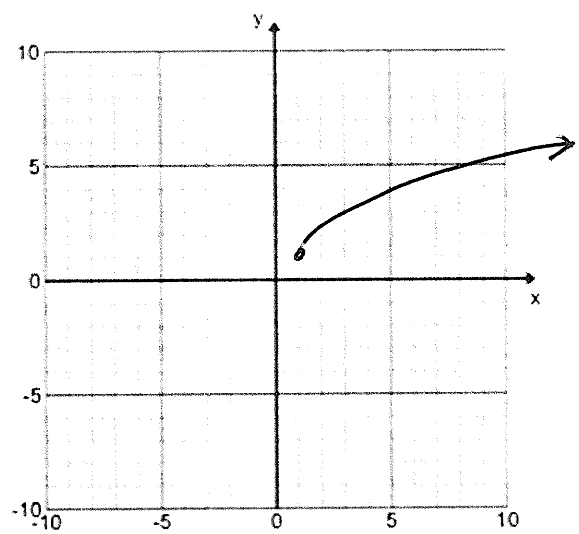
$5\sqrt{5}$

20. Write a function in the form of $y = a\sqrt{x-h} + k$ and has a graph that meets the condition given. Then sketch the graph for your function.

a. The graph is in the first quadrant only.

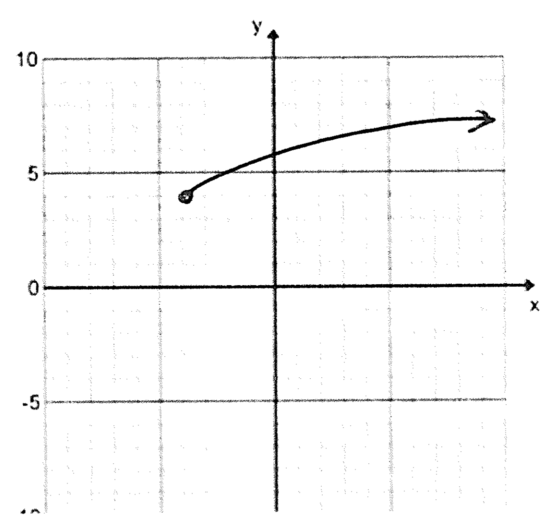
$y = a\sqrt{x-h} + K$

positive



b. The graph is in the first and second quadrants only.

$y = a\sqrt{x+h} + K$

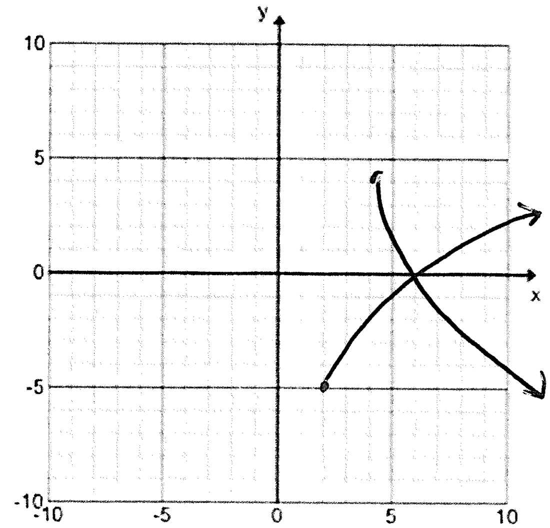


c. The graph is in the first and fourth quadrants only.

$y = -a\sqrt{x-h} + k$

$\boxed{\text{or}}$

$y = a\sqrt{x-h} - k$



COMPETENCY 3: RATIONAL EXPRESSIONS AND FUNCTIONS

Simplify each expression. State any excluded values.

1. $\frac{6x-48}{2x-16}$

$= \frac{6(x-8)}{2(x-8)} = \frac{6}{2} = 3$

$x \neq 8$

2. $\frac{3b^2+13b+4}{b+4}$

$\frac{(b+4)(3b+1)}{b+4} = 3b+1$

$b \neq -4$

3. $\frac{6-2x}{2x-6}$ $= \frac{-2(-3+x)}{2(x-3)} = \frac{-2}{2} = -1$

$x \neq 3$

Multiply or divide.

4. $\frac{2x+4}{x+2} \cdot \frac{3x}{4x+1}$

$= \frac{2(x+2)}{x+2} \cdot \frac{3x}{4x+1}$

$= \frac{2}{1} \cdot \frac{3x}{4x+1}$

$= \frac{6x}{4x+1}$

5. $\frac{2n-1}{n^2-4} \div \frac{n^2}{n+2}$

$= \frac{2n-1}{(n-2)(n+2)} \div \frac{n^2}{n+2}$

$= \frac{2n-1}{(n-2)(n+2)} \cdot \frac{n+2}{n^2}$

$= \frac{2n-1}{n^2(n-2)}$ or $\frac{2n-1}{n^3-2n^2}$

-12

4a 2a 4a 2a

6 3 4 -2 -1

6. $\dfrac{4a^2+4a-3}{2a+3} \div \dfrac{2a-1}{a}$

$$\frac{(\cancel{2a+3})(\cancel{2a-1})}{\cancel{2a+3}} \cdot \frac{a}{\cancel{2a-1}}$$

a

7. $\dfrac{x^2+8x+15}{x-4} \bullet \dfrac{x^2-16}{2x+6}$

$$\frac{(x+5)(\cancel{x+3})}{\cancel{x-4}} \cdot \frac{(\cancel{x-4})(x+4)}{2(\cancel{x+3})} = \frac{(x+5)}{1} \cdot \frac{(x+4)}{2}$$

$$\frac{x^2+9x+20}{2}$$

Add or subtract.

8. $\dfrac{a}{a}\left(\dfrac{10}{a}\right) + \dfrac{12}{a^2}$

$$= \frac{10a}{a^2} + \frac{12}{a^2}$$

$$= \boxed{\frac{10a+12}{a^2}}$$

9. $\dfrac{(x-1)}{(x-1)}\left(\dfrac{5}{x}\right) + \left(\dfrac{3}{x-1}\right)\dfrac{x}{x}$

$$= \frac{5x-5}{x(x-1)} + \frac{3x}{x(x-1)}$$

$$= \frac{8x-5}{x(x-1)}$$

10. $\frac{4}{m} - \frac{1}{2-m}$

$= \frac{(2-m)}{(2\cdot m)}\left(\frac{4}{m}\right) - \left(\frac{1}{2-m}\right)\frac{m}{m}$

$= \frac{8-4m}{m(2-m)} \quad \frac{-m}{m(2-m)}$

$= \frac{-5m+8}{m(2-m)}$

11. $\frac{x^2-1}{x^2-x-2} - \frac{(x-1)}{x-2}$

$= \frac{(x-1)\cancel{(x+1)}}{(x-2)\cancel{(x+1)}} \quad \frac{-x+1}{x-2}$

$= \frac{x-1}{x-2} \quad \frac{-x+1}{x-2}$

$= \frac{0}{x-2}$

$= 0$

Divide.

12. $(18x^3 + 12x^2 - 3x) \div 6x^2$

$= \frac{18x^3 + 12x^2 - 3x}{6x^2}$

$= \frac{18x^3}{6x^2} + \frac{12x^2}{6x^2} - \frac{3x}{6x^2}$

$= \boxed{3x + 2 - \frac{1}{2x}}$

13. $n-1 \overline{)5n^3 - 2n^2 + 4n - 10}$

$5n^2 + 3n + 7 + \frac{-3}{n-1}$

$-\ (5n^3 - 5n^2)$

$3n^2 + 4n$

$-\ (3n^2 - 3n)$

$7n - 10$

$-\ (7n - 7)$

-3

Solve each equation. Check your solutions.

14. $5y\left(\frac{1}{5}+\frac{2}{y}\right)=(1)5y$

$y + 10 = 5y$

$-y \qquad -y$

$10 = 4y$

$\frac{10}{4} = \frac{4y}{4}$

$\frac{5}{2} = y$

Check:

$\frac{1}{5} + \frac{2}{5/2} = 1$

$\frac{1}{5} + \frac{2}{1}\cdot\frac{2}{5} = 1$

$\frac{1}{5} + \frac{4}{5} = 1$ ✓

15. $2x\left(\frac{-6}{x}\right)=2x(4)+\left(\frac{x}{2}\right)2x$

$-12 = 8x + x^2$

$0 = x^2 + 8x + 12$

$0 = (x+6)(x+2)$

$x = -6 \quad x = -2$

Check:

$\frac{-6}{-6} = 4 + \frac{-6}{2}$

$1 = 4 - 3$ ✓

Check:

$\frac{-6}{-2} = 4 + \frac{-2}{2}$

$3 = 4 - 1$ ✓

16. $\frac{d}{d-1}=\frac{4d}{3d-2}$

$4d(d-1) = d(3d-2)$

$4d^2 - 4d = 3d^2 - 2d$

$-3d^2 + 2d \quad -3d^2 + 2d$

$d^2 - 2d = 0$

$d(d-2) = 0$

$d = 0 \quad d = 2$

Check:

$\frac{0}{0-1} = \frac{4(0)}{3(0)-2}$

$\frac{0}{-1} = \frac{0}{-2}$

$0 = 0$ ✓

$\frac{2}{2-1} = \frac{4(2)}{3(2)-2}$

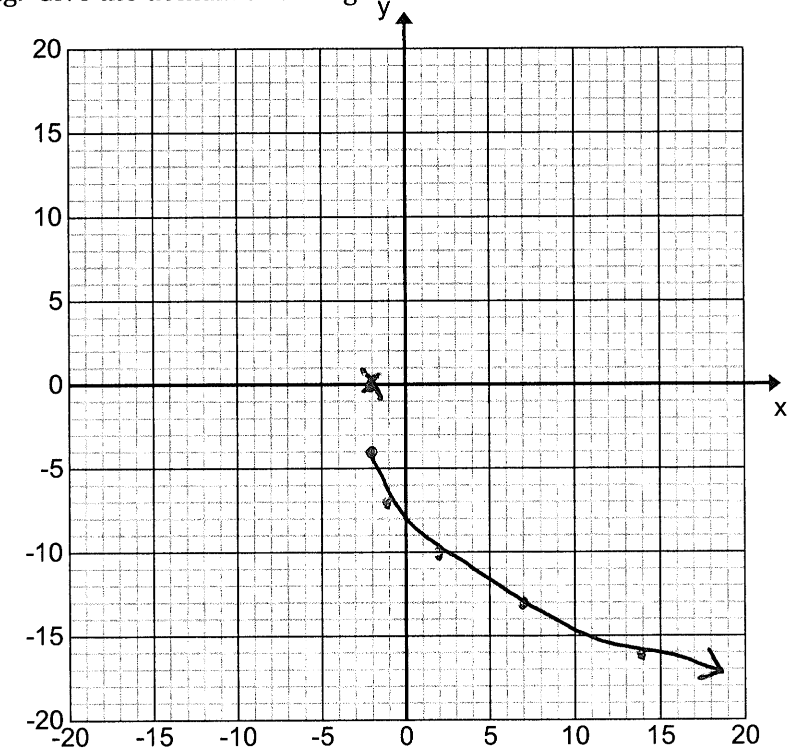
$\frac{2}{1} = \frac{8}{6-2}$

$2 = \frac{8}{4}$

$2 = 2$ ✓

All 3 competencies: Graph the following. Give the domain and range of each.

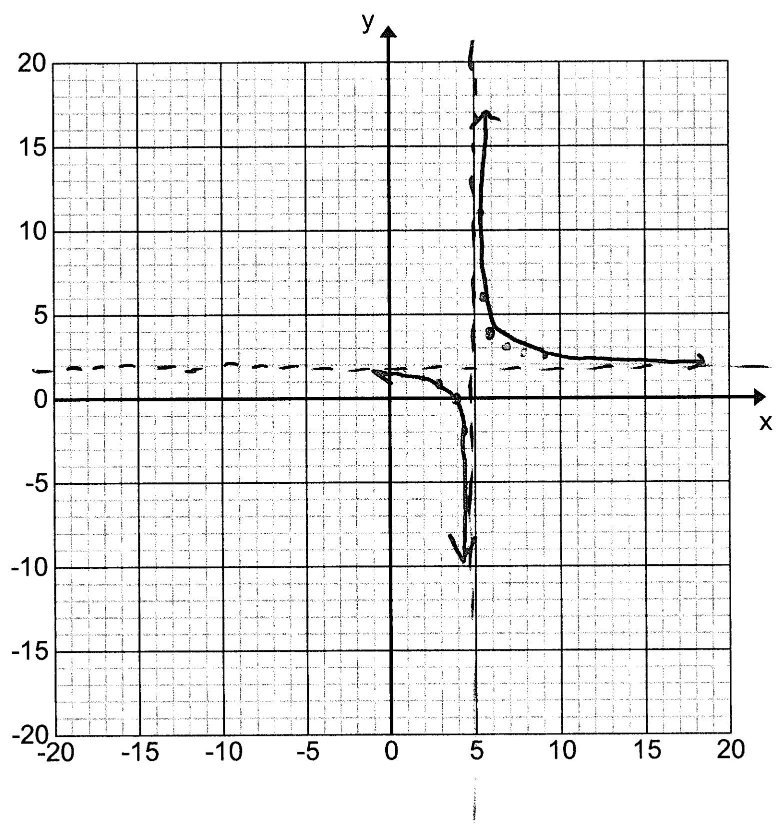
17. $y = -3\sqrt{x+2} - 4$



D: $x \geq -2$

R: $y \leq -4$

18. $f(x) = \dfrac{2}{x-5} + 2$



* Don't need to do

D: ℝ except $x = 5$

R: ℝ except $y = 2$

19. $y=\frac{1}{2}(x-4)^2-2$