Name ______________________________ Date__________

Algebra II -2 Final Exam Review-Part 1 (Chapter 6)

Simplify each radical expression. Rationalize all denominators.

1. $\dfrac{\sqrt{12x^7}}{\sqrt{3x^5}}$

$= \sqrt{4x^2}$

$= 2x$

2. $16^{\frac{5}{2}}$

$= (\sqrt{16})^5$

$= 4^5$

$= 1024$

3. 3^{-3}

$= \dfrac{1}{3^3}$

$= \dfrac{1}{27}$

4. $\sqrt[3]{27x^4y^5} \bullet \sqrt[3]{\overset{8\cdot 2}{16}x^7y}$

$= 3xy\sqrt[3]{xy^2} \cdot 2x^2\sqrt[3]{2xy}$

$= 6x^3y\sqrt[3]{2x^2y^3}$

$= 6x^3y\cdot y\sqrt[3]{2x^2}$

$= \boxed{6x^3y^2\sqrt[3]{2x^2}}$

5. $\sqrt{18} - \sqrt{50} + 4\sqrt{8}$

$9\cdot 2 \quad 25\cdot 2 \quad 4\cdot 2$

$3\sqrt{2} - 5\sqrt{2} + 4(2)\sqrt{2}$

$3\sqrt{2} - 5\sqrt{2} + 8\sqrt{2}$

$-2\sqrt{2} + 8\sqrt{2}$

$6\sqrt{2}$

6. $\dfrac{45x^8y^5z^{-2}}{12x^3y^{-2}z^5}$

$= \boxed{\dfrac{15x^5y^7}{4z^{7}}}$

7. $\left(4x^{-3}y^2\right)^{-2}\left(4x^4y^8\right)^{\frac{1}{2}}$

$4^{-2}x^6y^{-4}\cdot 4^{\frac{1}{2}}x^2y^4$

$\dfrac{4^{\frac{1}{2}}x^8}{4^2} = \dfrac{2x^8}{16} = \dfrac{x^8}{8}$

Simplify each radical expression. Rationalize all denominators.

8. $\frac{-4}{(3+\sqrt{5})} \cdot \frac{(3-\sqrt{5})}{(3-\sqrt{5})}$

$= \frac{-12+4\sqrt{5}}{9-5}$

$= \frac{-12+4\sqrt{5}}{4}$

$= -3+\sqrt{5}$

9. $(2\sqrt{2x})(2\sqrt{16x^3} - \sqrt{5x})$

$(2\sqrt{2x})(2(4)x\sqrt{x} - \sqrt{5x})$

$(2\sqrt{2x})(8x\sqrt{x} - \sqrt{5x})$

$16x\sqrt{2x^2} - 2\sqrt{10x^2}$

$16x \cdot x\sqrt{2} - 2x\sqrt{10}$

$16x^2\sqrt{2} - 2x\sqrt{10}$

Solve the following rational equations. Check for extraneous solutions.

10. $\sqrt{3x-2}^{\,2} = (x-2)^2$

$3x-2 = (x-2)(x-2)$

$3x-2 = x^2-4x+4$

$-3x+2 \qquad -3x+2$

$0 = x^2-7x+6$

$0 = (x-6)(x-1)$

$x=6 \qquad x=1$

Extraneous Solution

Check:

$\sqrt{3(6)-2} = 6-2$

$\sqrt{16} = 4$

$4 = 4$ ✓

$\sqrt{3(1)-2} = 1-2$

$\sqrt{3-2} = -1$

$1 = -1$ NO

11. $\left((2x-3)^{\frac{3}{2}}\right)^{2/3} = (27)^{2/3}$

$2x-3 = \left(\sqrt[3]{27}\right)^2$

$2x-3 = 3^2$

$2x-3 = 9$

$+3 \qquad +3$

$\frac{2x}{2} = \frac{12}{2}$

$x=6$

Check:

$(2(6)-3)^{3/2} = 27$

$(12-3)^{3/2} = 27$

$(\sqrt{9})^3 = 27$

$3^3 = 27$

$27 = 27$ ✓

12. In the expression $\sqrt[3]{8}$, 8 is called the Radicand.

13. In the expression $\sqrt[3]{8}$, 3 is called the Index.

14. Give an example of two radical expressions that are **NOT** like radicals (meaning they cannot be combined).

$\sqrt{x}$ and $\sqrt[3]{x}$ | $\sqrt{2}$ and $\sqrt{7}$

* there are many examples.

Find the inverse of the function. Graph the original function and its inverse. Find the domain and range of the original function and its inverse. Is the inverse a function?

15. $y = (x+2)^2 + 3$

PF $= x^2$, left 2, up 3

x	$y = x^2$
1	1
2	4
3	9
4	16

Inverse:

$x = (y+2)^2 + 3$

$-3 \qquad -3$

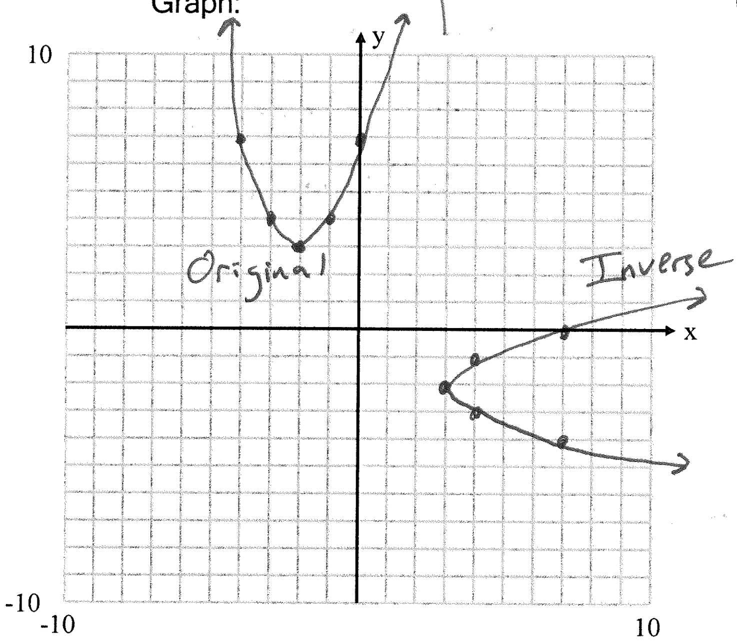
$\sqrt{x-3} = \sqrt{(y+2)^2}$

$\pm\sqrt{x-3} = y+2$

$-2 \qquad -2$

$\pm\sqrt{x-3} - 2 = y^{-1}$

Graph:



Domain/Range of both original/inverse:

Original

D: $\mathbb{R}$

R: $y \geq 3$

Inverse:

D: $x \geq 3$

R: $\mathbb{R}$

Is the inverse a function?

No → does not pass the vertical line test

16. $y = \sqrt{x-2} + 3$

PF $= \sqrt{x}$, Right 2, up 3

x	$\sqrt{x} = y$
1	1
4	2
9	3
16	4

Inverse:

$x = \sqrt{y-2} + 3$

$-3 \qquad -3$

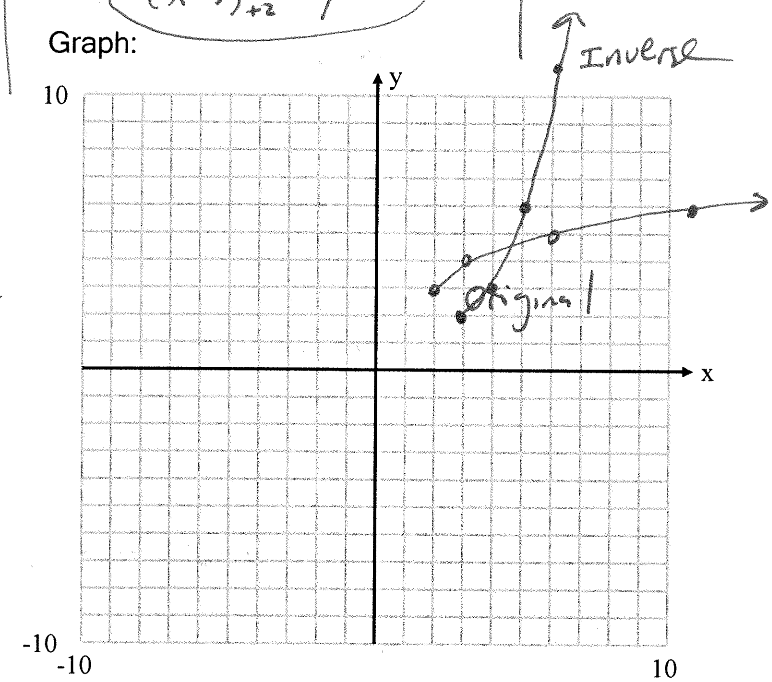
$(x-3)^2 = \sqrt{y-2}^{\,2}$

$(x-3)^2 = y-2$

$+2 \qquad +2$

$(x-3)^2 + 2 = y^{-1}$

Graph:



Domain/Range of both original/inverse:

Original:

D: $x \geq 2$

R: $y \geq 3$

Inverse:

D: $x \geq 3$

R: $y^{-1} \geq 2$

Is the inverse a function?

Yes → passes vertical line test

17. Let $f(x)=3x-9$ *and* $g(x)=x^2-3x$. Find the following as well as the domain of each.

a) $(f\circ g)(2) = f(g(2))$

$g(2) = (2)^2 - 3(2)$

$g(2) = 4 - 6$

$g(2) = -2$

$f(g(2)) = f(-2) = 3(-2) - 9 = -6 - 9 = -15$

Domain: $\mathbb{R}$

b) $(g\circ f)(x) = g(f(x)) = g(3x-9)$

$g(3x-9) = (3x-9)^2 - 3(3x-9)$

$= (3x-9)(3x-9) - 9x + 27$

$= 9x^2 - 54x + 81 - 9x + 27$

$= 9x^2 - 63x + 108$

Domain: $\mathbb{R}$

c) $\dfrac{g(x)}{f(x)} = \dfrac{x^2-3x}{3x-9} = \dfrac{x(\cancel{x-3})}{3(\cancel{x-3})} = \dfrac{x}{3}$

Domain: $\mathbb{R}$, but $x \neq 3$

d) $2g(x)+f(x)$

$2(x^2-3x) + 3x - 9$

$2x^2 - 6x + 3x - 9$

$2x^2 - 3x - 9$

Domain: $\mathbb{R}$

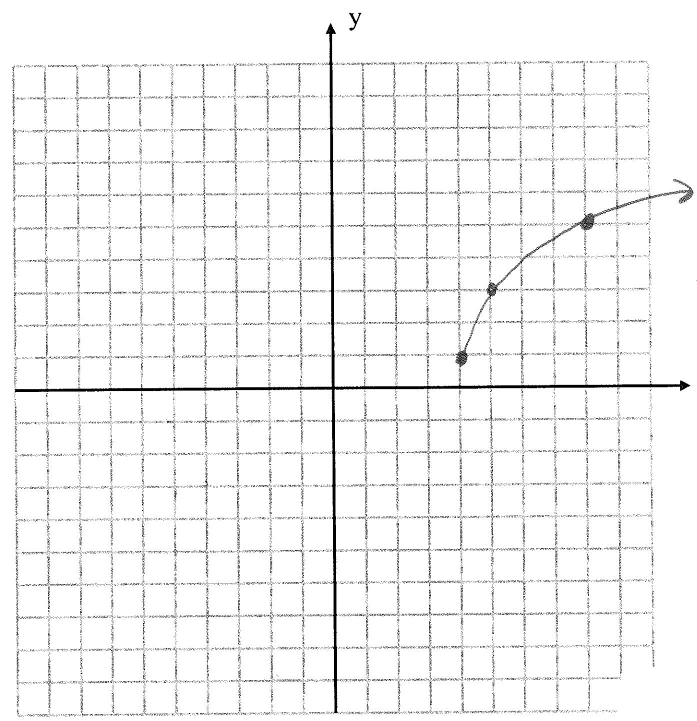
18. State the new origin, stretch, and graph the function. Then, give the domain and range.

$f(x)=2\sqrt{x-4}+1$ ~~left~~ Right 4, up 1

New Origin: (4, 1)

Stretch: $a = 2$

x	$\sqrt{x}=y$	$2\sqrt{x}$
1	1	2
4	2	4
9	3	6
16	4	8



Domain/Range:

$x \geq 4$ $\quad f(x) \geq 1$

Simplify each expression as much as possible, AND write your final answer in *radical form*.

19. $\left(a^{\frac{2}{3}}\right)^{\frac{5}{5}} \bullet \left(a^{\frac{1}{5}}\right)^{\frac{3}{3}}$

$= a^{\frac{10}{15}} \cdot a^{\frac{3}{15}}$

$= a^{\frac{13}{15}}$

$= \boxed{\sqrt[15]{a^{13}} \text{ or } = \left(\sqrt[15]{a}\right)^{13}}$

20. $\dfrac{\left(x^{\frac{2}{3}}y^{\frac{1}{2}}\right)^{\frac{2}{2}}}{\left(x^{\frac{1}{2}}\right)^{\frac{3}{3}}y^{\frac{1}{4}}} = \dfrac{x^{\frac{4}{6}}\, y^{\frac{2}{4}}}{x^{\frac{3}{6}}\, y^{\frac{1}{4}}}$

$= x^{\frac{1}{6}}\, y^{\frac{1}{4}}$

$= \sqrt[6]{x}\ \sqrt[4]{y}$

Simplify each radical expression as much as possible.

21. $\left(\sqrt{x}-\sqrt{3}\right)\left(\sqrt{x}+2\sqrt{3}\right)$

$x + 2\sqrt{3x} - \sqrt{3x} - 2(3)$

$\boxed{x + \sqrt{3x} - 6}$

22. $2\sqrt{32x^2} + 3\sqrt{72x^2}$

$16 \cdot 2 \qquad 36 \cdot 2$

$2(4)x\sqrt{2} + 3(6)x\sqrt{2}$

$8x\sqrt{2} + 18x\sqrt{2}$

$26x\sqrt{2}$