6-1b Notes: Roots and Radical Expressions

Lesson Objective: To understand how to simplify radical expressions with different indexes

radical symbol or radical

$$\sqrt[n]{a}$$

index radicand

	Perfect Squares	Perfect Cubes	Perfect Fourths
2	4	8	16
3	9	27	81
4	16	64	256
5	25	125	625
6	36	216	1296
7	49	343	2401
8	64	512	4096
9	81	729	6561
10	100	1000	10,000

Simplifying with radicals

Even and odd indexes

$\sqrt[4]{16} =$ 2

odd index, so negative radicand is cool

$\sqrt[3]{-27} =$ -3

-3(-3)(-3)

9(-3)

-27

$-\sqrt[4]{16} = -1(2)$

$= -2$

$-\sqrt[3]{-27} = -1(-3)$

$= 3$

$\sqrt{49} = 7$

$\sqrt[3]{0.008}$

$= \sqrt[3]{\frac{8}{1000}}$

$= \frac{2}{10}$

$= \frac{1}{5}$

$\sqrt[3]{-1000} = -10$

$\sqrt[3]{\frac{1}{27}} = \frac{1}{3}$

$\sqrt[4]{1} = 1$

With even indexes, negative radicands are bad

$\sqrt[4]{-.0001} = \sqrt[4]{-\frac{1}{10{,}000}}$

$= \frac{1}{10}i$

$\sqrt[4]{\frac{16}{81}} = \frac{2}{3}$

Find all the real square roots of each number.

1. 225 — 15

2. 0.0049 — $\sqrt{\frac{49}{10{,}000}}$ — $\frac{7}{100}$

3. $-\frac{1}{121}$ — $\frac{1}{11}i$

4. $\frac{64}{169}$ — $\frac{8}{13}$

Find all the real cube roots of each number.

5. -64 — -4

6. 0.125 — $\sqrt[3]{\frac{125}{1000}}$ — $\frac{5}{10}$ — $\frac{1}{2}$

7. $-\frac{27}{216}$ — $\frac{-3}{6}$ — $-\frac{1}{2}$

8. 0.000343 — $\sqrt[3]{\frac{343}{1{,}000{,}000}}$ — $\frac{7}{100}$

Find all the real fourth roots of each number.

9. 16 **10.** -16 **11.** 0.0081 **12.** $\frac{10,000}{81}$

2 $2i$ $\sqrt[4]{\frac{81}{10000}}$ $\frac{10}{3}$

$\frac{3}{10}$

Find each real-number root.

13. $\sqrt{36}$ **14.** $-\sqrt{36}$ **15.** $\sqrt{-36}$ **16.** $\sqrt{0.36}$

6 -6 $6i$ $\frac{36}{100}$

$\frac{6}{10}$

$\frac{3}{5}$

17. $-\sqrt[3]{64}$ **18.** $\sqrt[3]{-64}$ **19.** $-\sqrt[4]{81}$ **20.** $\sqrt[4]{-81}$

-4 -4 -3 $3i$

Academics Some teachers adjust test scores when a test is difficult. One teacher's formula for adjusting scores is $A = 10\sqrt{R}$, where A is the adjusted score and R is the raw score. If the raw scores on one test range from 36 to 90, what is the range of the adjusted scores?

Simplifying Radicals with Variables

$\sqrt{4x^6} = 2x^3$

Why? $2x^3 \cdot 2x^3 = 4x^6$

$\sqrt[3]{a^3b^6} = ab^2$

$\sqrt[4]{x^4y^5}$

$y^4 \cdot y^1$

$\sqrt[4]{x^4y^4y}$

$xy\sqrt[4]{y}$

$\sqrt[3]{x^6} =$

x^2

$\sqrt[4]{162x^9} =$

$81 \cdot 2 \quad x^8 \quad x^1$

$\sqrt[4]{81 \cdot 2 \cdot x^8 \cdot x}$

$3x^2\sqrt[4]{2x}$

$y^6 \cdot y^1$

$\sqrt{x^{12}y^7} = x^6y^3\sqrt{y}$

Simplify each radical expression.

21. $\sqrt{16x^2}$

$4x$

22. $\sqrt{0.25x^6}$

$\sqrt{\frac{25}{100}x^6}$

$\frac{5}{10}x^3$

$.5x^3$

$\frac{1}{2}x^3$

23. $\sqrt{x^8y^{18}}$

x^4y^9

24. $\sqrt{64b^{48}}$

$8b^{24}$

25. $\sqrt[3]{-64a^3}$

$-4a$

26. $\sqrt[3]{27y^6}$

$3y^2$

27. $\sqrt[4]{x^8y^{12}}$

x^2y^3

28. $\sqrt[5]{32y^{10}}$

$2y^2$

Simplify.

9. $\sqrt{20x^3}$ 10. $\sqrt[3]{81x^2}$ 11. $\sqrt{50x^5}$ 12. $\sqrt[3]{32a^5}$
13. $\sqrt[3]{54y^{10}}$ 14. $\sqrt{200a^6b^7}$ 15. $\sqrt[3]{-250x^6y^5}$ 16. $\sqrt[4]{64x^3y^6}$

(9) $\sqrt{20x^3}$; $4\cdot5$, $x^2\cdot x$; $2x\sqrt{5x}$

(10) $\sqrt[3]{81x^2}$; $27\cdot3$; $3\sqrt[3]{3x^2}$

(15) $\sqrt[3]{-250x^6y^5}$; $-125\cdot2$, y^3y^2; $-5x^2y\sqrt[3]{2y^2}$

(11) $\sqrt{50x^5}$; $25\cdot2$, $x^4\cdot x$; $5x^2\sqrt{2x}$

(12) $\sqrt[3]{32a^5}$; $8\cdot4$, a^3a^2; $2a\sqrt[3]{4a^2}$

(13) $\sqrt[3]{54y^{10}}$; $27\cdot2$, y^9y^1; $3y^3\sqrt[3]{2y}$

(14) $\sqrt{200a^6b^7}$; $100\cdot2$, $b^6\cdot b^1$; $10a^3b^3\sqrt{2b}$

(16) $\sqrt[4]{64x^3y^6}$; $16\cdot4$, y^4y^2; $2y\sqrt[4]{4x^3y^2}$

6-2 Notes: Multiplying and Dividing Radical Expressions

Lesson Objective: To understand how to simplify radical expressions after multiplying or dividing two separate expressions.

Product Property:

$\sqrt{4}\sqrt{25}$ $\sqrt[3]{-8}\sqrt[3]{27}$

Simplify:

$\sqrt[3]{4}\sqrt[3]{2}$

$\sqrt{-2}\sqrt{8}$

$\sqrt[3]{-5x}\sqrt[3]{25x}$

$\sqrt[3]{54x^2y^3}\sqrt[3]{5x^3y^4}$

Multiply, if possible. Then simplify.

1. $\sqrt{8} \cdot \sqrt{32}$
2. $\sqrt[3]{4} \cdot \sqrt[3]{16}$
3. $\sqrt[3]{9} \cdot \sqrt[3]{-81}$
4. $\sqrt[4]{8} \cdot \sqrt[4]{32}$
5. $\sqrt{-5} \cdot \sqrt{5}$
6. $\sqrt[3]{-5} \cdot \sqrt[3]{-25}$
7. $\sqrt[3]{9} \cdot \sqrt[3]{-24}$
8. $\sqrt[3]{-12} \cdot \sqrt[3]{-18}$

Quotient Property

Simplify: Divide then simplify

Indexes need to be equal

$$\frac{\sqrt{100}}{\sqrt{25}} = \frac{10}{5} = 2$$

$$= \sqrt{\frac{100}{25}} = \sqrt{4} = 2$$

$$\frac{\sqrt[3]{64}}{\sqrt[3]{8}} = \frac{4}{2} = 2$$

$$= \sqrt[3]{\frac{64}{8}} = \sqrt[3]{8} = 2$$

$$\frac{\sqrt[3]{32}}{\sqrt[3]{-4}} = \sqrt[3]{\frac{32}{-4}} = \sqrt[3]{-8} = -2$$

$$\frac{\sqrt[3]{162x^5y^4}}{\sqrt[3]{3x^2y}}$$

$27 \cdot 2$

$$\sqrt[3]{54x^3y^3}$$

$$3xy\sqrt[3]{2}$$

An radical expression is simplified when:

1) The coefficient (number) is broken down or simplified as much as possible.

2) All variables are simplified as much as possible (the exponents under radical are smaller than index).

3) No radicals left in denominator.

Rationalizing the Denominator: Get rid of radical in denominator.

$$\frac{\sqrt{5}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{15}}{\sqrt{9}} = \frac{\sqrt{15}}{3}$$

$$\frac{3}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$$

Simplify first, then rationalize any denominators.

$$\frac{\sqrt{6}}{\sqrt{5xy}} \cdot \frac{\sqrt{5xy}}{\sqrt{5xy}} \to \sqrt{25x^2y^2}$$

$$= \frac{\sqrt{30xy}}{5xy}$$

$$\frac{\sqrt[3]{2}}{\sqrt[3]{3x}} \cdot \frac{\sqrt[3]{3^2x^2}}{\sqrt[3]{3^2x^2}}$$

$$= \frac{\sqrt[3]{2 \cdot 9x^2}}{\sqrt[3]{3^3x^3}}$$

$$= \frac{\sqrt[3]{18x^2}}{3x}$$

$$\frac{2\sqrt[4]{2x^{\cancel{5}4}\cancel{y}}}{\sqrt[4]{3\cancel{x}y^{\cancel{3}2}}}$$

$$\frac{2\sqrt[4]{2x^4}}{\sqrt[4]{3y^2}}$$

$$\frac{2x\sqrt[4]{2}}{\sqrt[4]{3y^2}} \cdot \frac{\sqrt[4]{3^3y^2}}{\sqrt[4]{3^3y^2}} = \frac{2x\sqrt[4]{2(27)y^2}}{\sqrt[4]{3^4y^4}}$$

$$= \frac{2x\sqrt[4]{54y^2}}{3y}$$

Multiply and simplify. Assume that all variables are positive.

17. $\sqrt[3]{6} \cdot \sqrt[3]{16}$

18. $\sqrt{8y^5} \cdot \sqrt{40y^2}$

19. $\sqrt{7x^5} \cdot \sqrt{42xy^9}$

20. $4\sqrt{2x} \cdot 5\sqrt{6xy^2}$

21. $3\sqrt[3]{5y^3} \cdot 2\sqrt[3]{50y^4}$

22. $-\sqrt[3]{2x^2y^2} \cdot 2\sqrt[3]{15x^5y}$

odds

Rationalize the denominator of each expression.

odds

27. $\frac{\sqrt{x}}{\sqrt{2}}$ 28. $\frac{\sqrt{5}}{\sqrt{8x}}$ 29. $\frac{\sqrt[3]{x}}{\sqrt[3]{2}}$ 30. $\sqrt[3]{\frac{5}{3x}}$

31. $\frac{\sqrt[4]{2}}{\sqrt[4]{5}}$ 32. $\frac{15\sqrt{60x^5}}{3\sqrt{12x}}$ 33. $\frac{\sqrt{3xy^2}}{\sqrt{5xy^3}}$ 34. $\frac{\sqrt{5x^4y}}{\sqrt{2x^2y^3}}$

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