7-5 Notes: Solving Exponential and Logarithmic Equations

Lesson Objective: To find *exact* answers to exponential functions using *logarithms.*

45. The population of a certain animal species decreases at a rate of 3.5% per year. You have counted 80 of the animals in the habitat you are studying.

a. Write a function that models the change in the animal population.

b. ~~Graph the function. Estimate~~ Find the number of years until the population first drops below 15 animals.

a) $y = 80(1 - .035)^x$

b) T.B.C...

$15 = 80(1 - .035)^x$ ← Solve for x

An investment of $2300 earns 9.2% interest, which is compounded continuously. After how many years will the investment be worth $50,000?

$50{,}000 = 2300e^{.092t}$

T.B.C...

Question 1: How do we solve this equation? → Find like bases

→ the unknown is the exponent

$9^x = 27$

$(3^2)^x = 3^3$

$3^{2x} = 3^3$

$2x = 3$

$x = \frac{3}{2}$

Question 2: Can we solve the equation below by writing each side using the same base? NO

$2^x = 3^5$

$2^x = 243$

So...What *log* property will help us solve this equation?

$\log_b m^N = N \log_b m$

$\log 2^x = \log 3^5$

$\frac{x \cancel{\log 2}}{\cancel{\log 2}} = \frac{\log 243}{\log 2}$

$x = \frac{\log 243}{\log 2} = \log_2 243$ exact answer

$x \approx 7.925$ approximate answer

Examples: Solving Exponential Equations Using Logarithms

$7^{3x} = 20$

$\log 7^{3x} = \log 20$

$\frac{\cancel{3}x \cancel{\log 7}}{\cancel{3} \cancel{\log 7}} = \frac{\log 20}{3 \log 7}$

$x = \frac{1}{3}\log_7 20 = \log_7 \sqrt[3]{20}$

$= \log_7 (20)^{\frac{1}{3}}$

$x \approx 0.366$

$6^{2x} = 21$

$\log 6^{2x} = \log 21$

$\frac{2x \log 6}{2 \log 6} = \frac{\log 21}{2 \log 6}$

$x = \frac{\log 21}{2 \log 6} = \frac{1}{2}\log_6 21$

or

$\log_6 (21)^{\frac{1}{2}}$ or

$\log_6 \sqrt{21}$

$x \approx 0.514$

Examples: Solving Exponential Equations Using Logarithms

$3^{(x+4)} = 101$

$\log 3^{(x+4)} = \log 101$

$\frac{(x+4)\cancel{\log 3}}{\cancel{\log 3}} = \frac{\log 101}{\log 3}$

$x + 4 = \log_3 101$
$-4 \qquad -4$

$x = \log_3 101 - 4$

$x \approx \frac{\log 101}{\log 3} - 4$

$x \approx 0.201$

$5\left(\frac{2^{3x} - 7}{5}\right) = (100)\,5$

$2^{3x} - \cancel{7} = 500$
$+\cancel{7} \qquad +7$

$\log 2^{3x} = \log 507$

$\frac{\cancel{3}x\cancel{\log 2}}{\cancel{3}\cancel{\log 2}} = \frac{\log 507}{3\log 2}$

$x \approx 2.995$

$x = \frac{1}{3}\log_2 507$

Example: Suppose you invest P dollars at an annual interest rate of 4% compound continuously.

$Y = Pe^{rt}$

How long to triple your money?

$\frac{3\cancel{P}}{\cancel{P}} = \frac{\cancel{P}e^{.04t}}{\cancel{P}}$

$\log 3 = \log e^{.04t}$

$\frac{\log 3}{.04\log e} = \frac{\cancel{.04}t\cancel{\log e}}{\cancel{.04}\cancel{\log e}}$

$27.47 \approx t$ years

$.47(12) = 5.64$

27 years, 5.64 months

$.64(30) = 19.2$

27 yrs, 5 months, 19.2 days

How long to increase your investment 50%?

$\frac{1.5P}{P} = \frac{Pe^{.04t}}{P}$

$1.5 = e^{.04t}$

$\frac{\log 1.5}{.04\log e} = \frac{\cancel{.04}t\cancel{\log e}}{\cancel{.04}\cancel{\log e}}$

$10.14 \approx t$

10 years and 1.68 months $\approx t$

Example: At what rate would you invest money so you would double your investment in 12 years compounded quarterly?

$$\frac{2\cancel{P}}{\cancel{P}} = \frac{\cancel{P}\left(1+\frac{r}{4}\right)^{4(12)}}{\cancel{P}}$$

$$(2)^{\frac{1}{48}} = \left(\left(1+\frac{r}{4}\right)^{\cancel{48}}\right)^{\frac{1}{48}}$$

$$2^{\frac{1}{48}} = 1 + \frac{r}{4}$$

$$-1 \qquad -1$$

$$4\left(2^{\frac{1}{48}} - 1\right) = \left(\frac{r}{\cancel{4}}\right)\cancel{4}$$

Solving Logarithmic Equations

Examples: Solve the following equations → change to exponential.

$\log(3x+1) = 5$

$10^5 = 3x+1$ → Going to exponential

$100{,}000 = 3x+1$

$-1 \qquad -1$

$99{,}999 = 3x$

$33{,}333 = x$

$2\log x - \log 3 = 2$

1st need to write as a single log before converting to exponential

$\log x^2 - \log 3 = 2$

$\log\left(\frac{x^2}{3}\right) = 2$

$3\,(10^2) = \left(\frac{x^2}{\cancel{3}}\right)\cancel{3}$

$\sqrt{300} = \sqrt{x^2}$

$100 \cdot 3$

$10\sqrt{3} = x$

$17.32 \approx x$

Examples: Solve the following equations

$\log 6 - \log 3x = -2$

$\log \frac{6}{3x} = -2$

$10^{-2} = \frac{2}{x}$

~~$\frac{1}{10^2} = \frac{2}{x}$~~

$x = 2(100)$

$x = 200$

base = "e"

$\ln(3x+5)^2 = 4$

$\frac{\cancel{2}\ln(3x+5)}{\cancel{2}} = \frac{4}{2}$

$\ln(3x+5) = 2$

$e^2 = 3x+5$

$\frac{e^2 - 5}{3} = \frac{3x}{3}$

$\frac{e^2 - 5}{3} = x$

$.796 \approx x$

Example: Conservation efforts have increased the endangered Florida Manatee population from 1495 in 2007 to 3276 in 2017. If the annual growth rate continues when might there be 10,000 manatees?

$y = ab^x$

x = time

y = population of manatees

$(0, 1495)$

$(10, 3276)$