

**6-6 Notes: Function Operations**

Lesson Objective: To add, subtract, multiply, and divide two or more functions. Also, to find the composite of two functions.

**Function Notation:**

1) If  $f(x) = 3x + 4$  find  $f(2)$

$$f(2) = 3(2) + 4$$

$$f(2) = 6 + 4$$

$$f(2) = 10$$

2) If  $g(x) = \frac{2x^2 + 4}{6}$  find  $g(-6.5)$ . Round to nearest hundredth.

$$g(-6.5) = \frac{2(-6.5)^2 + 4}{6}$$

$$g(-6.5) = \frac{2(42.25) + 4}{6}$$

$$g(-6.5) = \frac{84.5 + 4}{6}$$

$$g(-6.5) = \frac{88.5}{6}$$

$$g(-6.5) = 14.75$$

**FUNCTION OPERATIONS**

Addition

$$(f+g)(x) = f(x) + g(x) = f + g$$

Multiplication

$$(f \cdot g)(x) = f(x) \cdot g(x) = f \cdot g$$

Subtraction

$$(f-g)(x) = f(x) - g(x) = f - g$$

Division

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{f}{g} \quad ; \quad g(x) \neq 0$$

Example: Let  $f(x) = 3x + 8$  and  $g(x) = 2x - 12$

Find  $f + g = f(x) + g(x)$   
 $= 3x + 8 + 2x - 12$   
 $= 5x - 4$

Domain  
 $\mathbb{R}$

$f - g = f(x) - g(x)$   
 $= 3x + 8 - (2x - 12)$   
 $= 3x + 8 - 2x + 12$   
 $= x + 20$

Domain  
 $\mathbb{R}$

$f - g = x + 20$

find  $f \cdot g = f(x) \cdot g(x)$   
 $= (3x + 8)(2x - 12)$   
 $= 6x^2 - 36x + 16x - 96$   
 $= 6x^2 - 20x - 96$

Domain  
 $\mathbb{R}$

find  $\frac{g}{f} = \frac{g(x)}{f(x)} = \frac{2x - 12}{3x + 8}$

Domain

$3x + 8 = 0$   
 $3x = -8$   
 $x = -\frac{8}{3}$

$(-\infty, \frac{8}{3}) \cup (-\frac{8}{3}, \infty)$

$\mathbb{R}$  except  $x \neq -\frac{8}{3}$

Example: Let  $f(x) = 6x^2 - 5$  and  $g(x) = 2x - 1$

$$f + g = 6x^2 - 5 + 2x - 1$$

$$= 6x^2 + 2x - 6$$

Domain

 $\mathbb{R}$ 

$$f - g = 6x^2 - 5 - (2x - 1)$$

$$= 6x^2 - 5 - 2x + 1$$

$$= 6x^2 - 2x - 4$$

Domain

 $\mathbb{R}$ 

$$f \cdot g = (6x^2 - 5)(2x - 1)$$

$$= 12x^3 - 6x^2 - 10x + 5$$

Domain

 $\mathbb{R}$ 

$$\frac{f}{g} = \frac{6x^2 - 5}{2x - 1}$$

Domain

$$2x - 1 = 0$$

$$2x = 1$$

$$x = \frac{1}{2}$$

 $\mathbb{R}$  except

$$x \neq \frac{1}{2}$$

Example: If:  $f(x) = 2x - 5$

Find the following:

$$\begin{aligned}
 & 3g(x) - 2f(x) \\
 &= 3(x^2 - 4) - 2(2x - 5) \\
 &= 3x^2 - 12 - 4x + 10 \\
 &= 3x^2 - 4x - 2
 \end{aligned}$$

and

$$g(x) = x^2 - 4$$

$$\begin{aligned}
 & -2g(x) + f(x) \\
 &= -2(x^2 - 4) + 2x - 5 \\
 &= -2x^2 + 8 + 2x - 5 \\
 &= -2x^2 + 2x + 3
 \end{aligned}$$

$$-3f(x) \cdot g(x)$$

$$\begin{aligned}
 & -3(2x - 5)(x^2 - 4) \\
 &= (-6x + 15)(x^2 - 4) \\
 &= -6x^3 + 24x + 15x^2 - 60 \\
 &= -6x^3 + 15x^2 + 24x - 60
 \end{aligned}$$

$$\begin{aligned}
 & x^2 - 4 = 0 \\
 & \sqrt{x^2} = \sqrt{4} \\
 & x = \pm 2
 \end{aligned}$$

$$\frac{2f(x)}{g(x)}$$

$$\begin{aligned}
 &= \frac{2(2x - 5)}{x^2 - 4} \\
 &= \frac{4x - 10}{x^2 - 4} \quad \text{or} \\
 &= \frac{4x - 10}{(x - 2)(x + 2)}
 \end{aligned}$$

$$\begin{aligned}
 & \downarrow \\
 & x - 2 = 0 \quad x + 2 = 0 \\
 & x = 2 \quad x = -2
 \end{aligned}$$

All reals except  $x = \pm 2$

COMPOSITE FUNCTIONS

Not a multiplication symbol.  
Composition symbol.

$(f \circ g)(x) = f(g(x))$  "f composed of g"  
Steps 1.

$(g \circ f)(x) = g(f(x))$  "g composed of f"

Substitute the "inside" function into x of the "outside" function.

2. Simplify as much as possible by combining like terms.

Example: Let  $f(x) = x - 2$  and

$g(x) = x^2$

find the following:  
Work inside to out.

$(g \circ f)(-5) = g(f(-5))$   
 $f(-5) = -5 - 2$   
 $f(-5) = -7$   
 $g(f(-5)) = g(-7)$   
 $g(-7) = (-7)^2$   
 $g(-7) = 49$   
 $g(f(a)) = g(f(-5)) = 49$

$f(a) = a - 2$   
 $g(f(a)) = g(a - 2)$   
 $g(a - 2) = (a - 2)^2$   
 $g(a - 2) = (a - 2)(a - 2)$   
 $g(a - 2) = a^2 - 4a + 4$   
 $g(f(a)) = a^2 - 4a + 4$

$(f \circ g)(-5) = f(g(-5))$   
 $g(-5) = (-5)^2$   
 $g(-5) = 25$   
 $f(g(-5)) = f(25)$   
 $f(25) = 25 - 2$   
 $f(25) = 23$   
 $f(g(-5)) = 23$   
 $(f \circ g)(x) = f(g(x))$

$f(g(x)) = f(x^2)$   
 $f(x^2) = x^2 - 2$   
 $f(g(x)) = x^2 - 2$

Example: Let  $f(x) = x^3 - 4$  and  $g(x) = x^2 + 5x$   
find the following:

$$(g \circ f)(-3)$$

$$(g \circ f)(-a)$$

### Homework

Let  $f(x) = 2x^2 + x - 3$  and  $g(x) = x - 1$ . Perform each function operation and then find the domain.

13.  $f(x) + g(x)$

14.  $g(x) - f(x)$

16.  $f(x) \cdot g(x)$

17.  $\frac{f(x)}{g(x)}$

Let  $f(x) = 3x + 5$  and  $g(x) = x^2$ . Perform each function operation, and state the domain

1.  $f(x) + g(x)$

2.  $g(x) - f(x)$

4.  $f(x) \cdot g(x)$

5.  $\frac{f(x)}{g(x)}$

Let  $f(x) = x^2$  and  $g(x) = x - 3$ . Find each value or expression.

31.  $(g \circ f)(-2)$

32.  $(f \circ g)(-2)$

33.  $(g \circ f)(0)$

34.  $f(g(x))$

35.  $g(f(x))$

Let  $f(x) = x^2$  and  $g(x) = x - 3$ . Find each value or expression.

40.  $(g \circ f)(c)$

41.  $(f \circ g)(-a)$

42.  $(g \circ f)(-a)$