Name____Date______________________

Algebra 2-1 Final Exam Review

Chapter 3 Material (Competency 1)

1. Solve the system of equation using the substitution method.

$$4x+3y=5$$
$$2x+y=7$$
$\hookrightarrow y=-2x+7$

$4x+3(-2x+7)=5$

$4x-6x+21=5$

$-2x+21=5$
$-21 \quad -21$

$\frac{-2x}{-2}=\frac{-16}{-2}$

$x=8$

$y=-2(8)+7$

$y=-16+7$

$y=-9$

$\boxed{(8,-9)}$

2. Solve the system of equations using the elimination method.

$5(3x+7y=15) \rightarrow 15x+35y=75$
$-3(5x+2y=-4) \quad + \quad -15x-6y=12$

$\frac{29y}{29}=\frac{87}{29}$

$y=3$

$3x+7(3)=15$

$3x+21=15$
$-21 \quad -21$

$\frac{3x}{3}=\frac{-6}{3}$

$x=-2$

$(-2,3)$

3. A landscaping company placed two orders with a nursery. The first order was for 13 bushes and 4 trees, and totaled \$487. The second order was for 6 bushes and 2 trees, and totaled \$232. What were the costs of one bush and of one tree? (Write and solve a system of equations and define your variables).

$x=$ cost of 1 bush
$y=$ " " 1 tree

$13x+4y=487$
$-2(6x+2y=232)$

$13x+4y=487$
$-12x-4y=-464$

$x=23$

$6(23)+2y=232$

$138+2y=232$
$-138 \quad -138$

$\frac{2y}{2}=\frac{94}{2}$

$y=47$

1 bush costs \$23 and 1 tree costs \$47

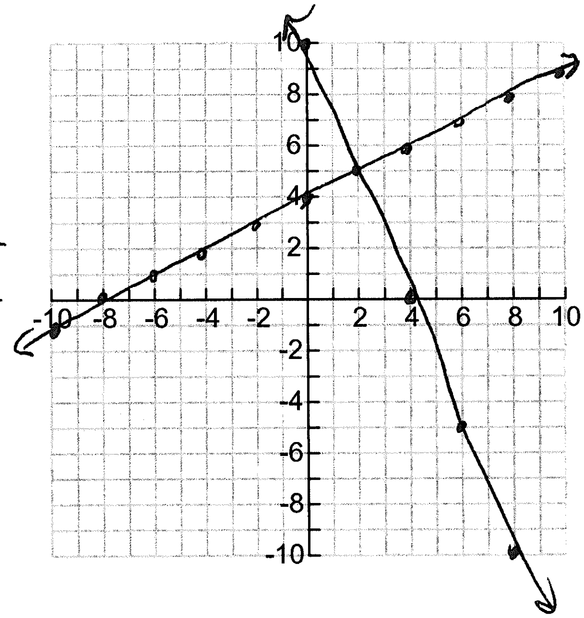
4. Solve the system of equations by graphing.

$5x + 2y = 20$ → $(4,0)$ and $(0,10)$ are the intercepts

$y = \frac{1}{2}x + 4$

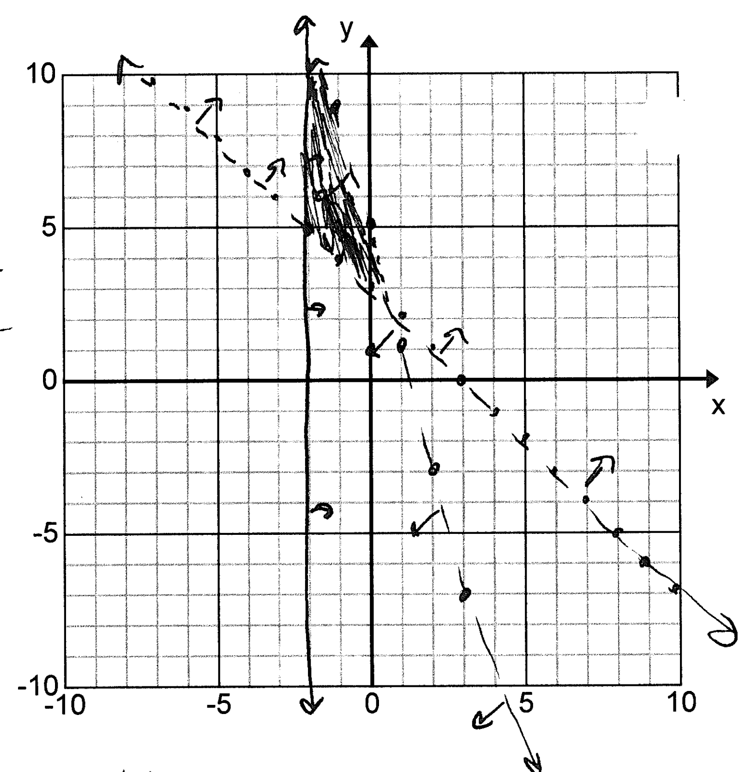
$m = \frac{-10}{4} = \frac{-5}{2}$

$(2,5)$



5. Graph the system of linear inequalities $\begin{array}{l} y < -4x+5 \\ y-5 > -(x+2) \\ x \geq -2 \end{array}$. Then give two solutions (x,y) to the system.

- For $y-5 > -(x+2)$, it's in point slope form, so the point is $(-2,5)$ and the slope is $m=-1$



Example Solutions: $(-1,5)$, $(-1,6)$, $(-2,6)$, $(-2,7)$, $(-2,8)$ etc.

6. Solve the 3-variable system of equations:

$$\begin{cases} x + 2y + z = 4 \\ 2x - y + 4z = -8 \\ -3x + y - 2z = -1 \end{cases}$$

① $+$

$-x + 2z = -9$

② $x + 2y + z = 4$

$(2x - y + 4z = -8)2$

$x + 2y + z = 4$

$+\ 4x - 2y + 8z = -16$

$5x + 9z = -12$

③ $(-x + 2z = -9)5$

$5x + 9z = -12$

$+\ -5x + 10z = -45$

$5x + 9z = -12$

$\frac{19z}{19} = \frac{-57}{19}$

$z = -3$

④ $-x + 2(-3) = -9$

$-x - 6 = -9$

$+6 \quad +6$

$\frac{-x}{-1} = \frac{-3}{-1}$

$x = 3$

⑤ $3 + 2y + -3 = 4$

$\frac{2y}{2} = \frac{4}{2}$

$y = 2$

$\boxed{(3, 2, -3)}$

7. A manufacturing company makes two type of water skis, a trick ski and a slalom ski. It takes 6 hours of labor to fabricate a pair of trick skis and 4 hours for a pair of slalom skis. After fabricating the skis, it then takes 1 hour to finish a pair of trick skis and 1 hour to finish a pair of slalom skis. There are 108 labor hours available per day for fabricating skis and 24 labor hours available for finishing them.

Department	Labor Hours Per Ski		Maximum Labor-Hours Available Per Day
	x Trick Ski	y Slalom Ski	
Fabricating	6 x +	4y	≤ 108
Finishing	1 x +	1 y	≤ 24
Profit	40x +	30 y	= MAXIMIZE

This info is on the next page

Continued →

Problem 7 continued...

If the profit on a trick ski is \$40 and the profit on a slalom ski is \$30, how many of each type of ski should be manufactured each day to realize a maximum profit? What is the maximum profit?

$6x + 4y \le 108$

$-6x \qquad -6x$

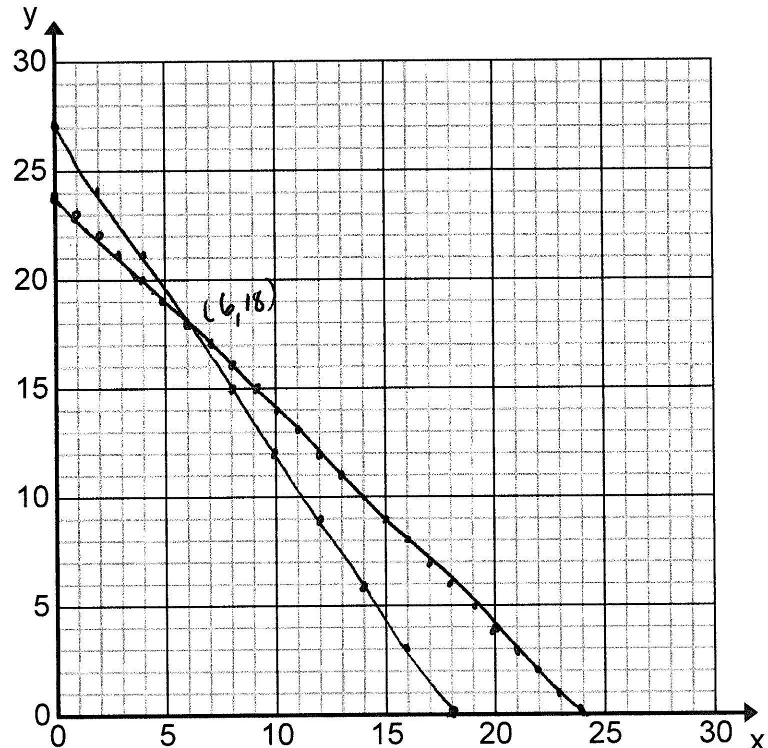
$\frac{4y}{4} \le \frac{-6x}{4} + \frac{108}{4}$

$y \le -\frac{3}{2}x + 27$

$x + y \le 24$

$-x \qquad -x$

$y \le -x + 24$



Corners: (0,24), (6,18), (18,0)

$P = 40(0) + 30(24) = 720$

$P = 40(6) + 30(18) = 780$

$P = 40(18) + 30(0) = 720$

The max profit is \$780, manufacturing 6 trick skis and 18 slalom skis.

Chapter 4 Material (competency 2):

Factor the following quadratics:

1. $3x^2 - 8x + 4$

$(x-2)(3x-2)$

2. $x^2 - 4x - 32$

$(x-8)(x+4)$

3. $6x - 18x^2$

GCF = $6x$ or $-6x$

$6x(1-3x)$

or

$-6x(-1+3x)$

4. $3x^2 + 15x - 42$

GCF = 3

$3(x^2 + 5x - 14)$

$3(x+7)(x-2)$

Solve by factoring

5. $2x^2 + 26x = 0$

GCF = $2x$

$2x(x + 13) = 0$

$\frac{2x}{2} = \frac{0}{2}$ $\quad x + 13 = 0$
$\quad -13 \; -13$

$x = 0$ $\quad x = -13$

6. $x^2 = 14x + 32$
$-14x - 32 \; -14x \; -32$

$x^2 - 14x - 32 = 0$

-32; -16, 2; -14

$(x - 16)(x + 2) = 0$

$x - 16 = 0$ $\quad x + 2 = 0$

$x = 16$ $\quad x = -2$

7. $2x^2 + x = 28$
$-28 \quad -28$

$2x^2 + x - 28 = 0$

-56; $2x$, x; 8, 4; 1; $2x$; -7

$(x + 4)(2x - 7) = 0$

$x + 4 = 0$ $\quad 2x - 7 = 0$
$\quad +7 \; +7$

$x = -4$ $\quad 2x = 7 \rightarrow x = 7/2$

8. $x^2 - 9 = 0$

-9; -3, $+3$; 0

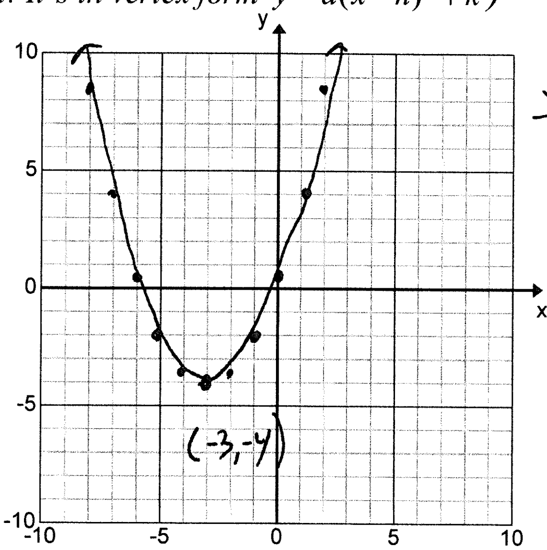
$(x - 3)(x + 3) = 0$

$x - 3 = 0$ $\quad x + 3 = 0$

$x = 3$ $\quad x = -3$

. Graph the equation $y = \frac{1}{2}(x+3)^2 - 4$

(Hint: It's in vertex form $y = a(x-h)^2 + k$)



x	$y = x^2$	$y = \frac{1}{2}x^2$
1	1	1/2
2	4	2
3	9	4.5
4	16	8
5	25	12.5
6	36	18

Domain and range: D: $\mathbb{R}$

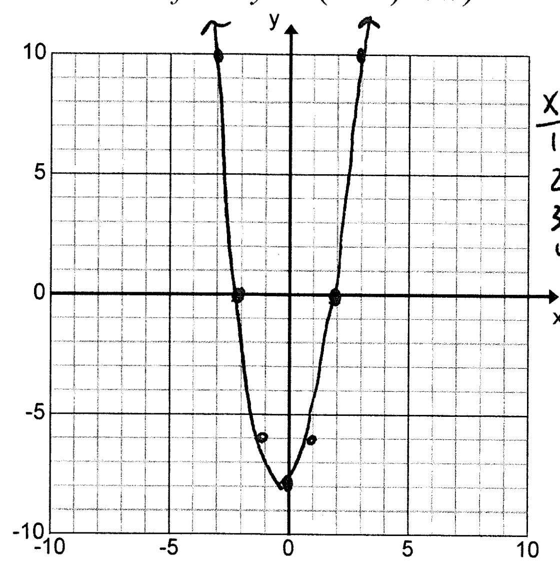
R: $y \geq -4$

...s of Symmetry:

$x = -3$

10. Graph the equation $y = 2x^2 - 8 = 2(x - 0)^2 - 8$

(Hint: It's in vertex form $y = a(x-h)^2 + k$)



x	$y = x^2$	$2x^2$
1	1	2
2	4	8
3	9	18
4	16	32

Domain and range: D: $\mathbb{R}$

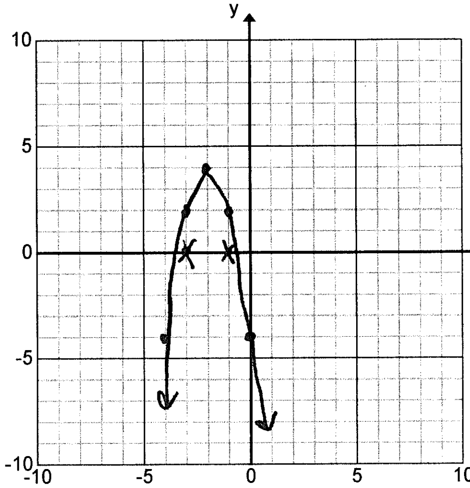
R: $y \geq -8$

Axis of Symmetry: $x = 0$

11. Graph the equation $y = -2x^2 - 8x - 4$

(Hint: It's in standard form $y = ax^2 + bx + c$ *,*

find the vertex using $x = \frac{-b}{2a}$ *)*



$x = \frac{8}{2(-2)} = \frac{8}{-4} = -2$

$y = -2(-2)^2 - 8(-2) - 4$

$y = -2(4) + 16 - 4$

$y = -8 + 16 - 4$

$y = 8 - 4$

$y = 4$

$(-2, 4) =$ vertex

x	$y=x^2$	$y=-2x^2$
1	1	-2
2	4	-8
3	9	-18
4	16	-32

Domain and range:

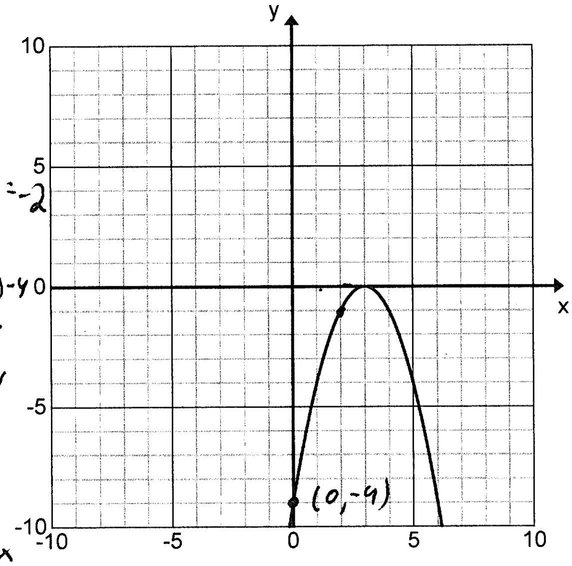
D: $\mathbb{R}$

R: $y \leq 4$

Min/Max and where?

max @ $(-2, 4)$

12. Give the equation of the quadratic.



Equation: Vertex = $(3, 0)$

$y = a(x-3)^2 + 0$

$-9 = a(0-3)^2 + 0 \rightarrow -9 = a(-3)^2$

$\frac{-9}{9} = \frac{a \cdot 9}{9}$

$-1 = a$

$y = -(x-3)^2$

Domain and range:

D: $\mathbb{R}$

R: $y \leq 0$

13. Given the equation $y = -3x^2 + 6x - 5$, find the min/max value. What is the domain and range?

vertex

$x = \frac{-6}{2(-3)} = \frac{-6}{-6} = 1$

$y = -3(1)^2 + 6(1) - 5$

$y = -3(1) + 6 - 5$

$y = -3 + 6 - 5$

$y = 3 - 5$

$y = -2$

Max value @ $y = -2$

$(1, -2)$

14. Suppose a model rocket is launched from a platform 2 ft above the ground with an initial upwards velocity of 100 ft/s.

a) Using the equation $h(t) = -16t^2 + v_0 t + h_0$, $v_0 = \textit{initial velocity}$ and $h_0 = \textit{initial height}$, write a function that represents the path of the rocket.

$h(t) = -16t^2 + 100t + 2$

b) When will the rocket reach its $\boxed{\text{maximum}}$ height?

vertex

$t = \frac{-b}{2a} = \frac{-100}{2(-16)} = \frac{-100}{-32} = 3.125$ seconds

c) What is the rocket's $\boxed{\text{maximum}}$ height?

vertex

$h(3.125) = -16(3.125)^2 + 100(3.125) + 2$

$= -16(9.765625) + 312.5 + 2$

$= -156.25 + 312.5 + 2$

$= 156.25 + 2$

158.25 ft is the max height

Problem 14 continued...

d) When will the rocket hit the ground?

$$0 = -16t^2 + 100t + 2$$

$$t = \frac{-100 \pm \sqrt{100^2 - 4(-16)(2)}}{2(-16)}$$

$$t = \frac{-100 \pm \sqrt{10{,}000 + 128}}{-16}$$

$$t = \frac{-100 \pm \sqrt{10{,}128}}{-16}$$

$$\frac{-100 \pm 100.64}{-16}$$

$$\frac{-100 + 100.64}{-16} \qquad \frac{.64}{-16} = \cancel{-.04}$$

$$\frac{-100 - 100.64}{-16} \qquad \frac{-200.64}{-16}$$

12.54

$\boxed{\text{12.54 seconds to hit ground}}$

15. Write the quadratic equation of the parabola that crosses through the points (2, 0), (3, -2), and (1, -2).

$y = ax^2 + bx + c$

① $0 = a(2)^2 + b(2) + c$

$0 = 4a + 2b + c$

② $-2 = a(3)^2 + b(3) + c$

$-2 = 9a + 3b + c$

③ $-2 = a(1)^2 + b(1) + c$

$-2 = a + b + c$

④ $-1(-2 = a + b + c)$

$$\begin{array}{rl} & 2 = -a - b - c \\ + & -2 = 9a + 3b + c \\ \hline & 0 = 8a + 2b \end{array}$$

⑤
$$\begin{array}{rl} & 2 = -a - b - c \\ + & 0 = 4a + 2b + c \\ \hline & 2 = 3a + b \end{array}$$

⑥ $(2 = 3a + b)(-2)$

$0 = 8a + 2b$

$$\begin{array}{rl} & -4 = -6a - 2b \\ + & 0 = 8a + 2b \\ \hline & -4 = 2a \end{array}$$

$\frac{-4}{2} = \frac{2a}{2}$

$-2 = a$

⑦ $2 = 3(-2) + b$

$2 = -6 + b$

$+6 \quad +6$

$8 = b$

⑧ $-2 = -2 + 8 + c$

$-2 = 6 + c$

$-6 \quad -6$

$-8 = c$

$y = -2x^2 + 8x - 8$

16. Solve the following equations by completing the square. Give your answers in *exact form*.

a. $x^2 + 8x = -6$

$\left(\frac{b}{2}\right)^2 = \left(\frac{8}{2}\right)^2 = (4)^2 = 16$

$x^2 + 8x + 16 = -6 + 16$

$\sqrt{(x+4)^2} = \sqrt{10}$

$x + 4 = \pm\sqrt{10}$

$-4 \qquad -4$

$x = \pm\sqrt{10} - 4$

b. $4x^2 - x - 3 = 0$

$\frac{4x^2}{4} - \frac{x}{4} - \frac{3}{4} = \frac{0}{4}$

$x^2 - \frac{1}{4}x - \frac{3}{4} = 0$

$x^2 - \frac{1}{4}x = \frac{3}{4}$

$\left(\frac{\frac{1}{4}}{2}\right)^2 = \left(\frac{1}{4} \cdot \frac{1}{2}\right)^2 = \left(\frac{1}{8}\right)^2 = \frac{1}{64}$

$x^2 - \frac{1}{4}x + \frac{1}{64} = \left(\frac{3}{4}\right)\frac{16}{16} + \frac{1}{64}$

$\left(x - \frac{1}{8}\right)^2 = \frac{48}{64} + \frac{1}{64}$

$\sqrt{\left(x - \frac{1}{8}\right)^2} = \sqrt{\frac{49}{64}}$

$x - \frac{1}{8} = \pm\frac{7}{8}$

$+\frac{1}{8} \qquad +\frac{1}{8}$

$x = \pm\frac{7}{8} + \frac{1}{8}$

$\frac{7}{8} + \frac{1}{8} \qquad -\frac{7}{8} + \frac{1}{8}$

$\frac{8}{8} \qquad -\frac{6}{8}$

$1 \qquad -3/4$

17. Solve the following using the quadratic formula. Round answers to the nearest *hundredth* if needed.

a. $5x^2 - 7x - 3 = 0$

$x = \frac{7 \pm \sqrt{(-7)^2 - 4(5)(-3)}}{2(5)}$

$x = \frac{7 \pm \sqrt{49 + 60}}{10}$

$x = \frac{7 \pm \sqrt{109}}{10}$

$x = \frac{7 \pm 10.44}{10}$

$x = \frac{7 + 10.44}{10}$ $\quad x = \frac{7 - 10.44}{10}$

$x = \frac{17.44}{10}$ $\quad x = \frac{-3.44}{10}$

$x = 1.74$ $\quad x = -.34$

b. $3x^2 + 3 = 6x$

$3x^2 - 6x + 3 = 0$

$x = \frac{6 \pm \sqrt{(-6)^2 - 4(3)(3)}}{2(3)}$

$x = \frac{6 \pm \sqrt{36 - 36}}{6}$

$x = \frac{6 \pm \sqrt{0}}{6}$

$x = \frac{6 \pm 0}{6}$

$x = \frac{6}{6}$

$x = 1$

18. Solve using square roots. Give exact answers.

a. $4x^2 - 400 = 0$

$+400 \quad +400$

$\frac{4x^2}{4} = \frac{400}{4}$

$\sqrt{x^2} = \sqrt{100}$

$x = \pm 10$

b. $-2(x+3)^2 - 2 = -12$

$+2 \quad +2$

$\frac{-2(x+3)^2}{-2} = \frac{-10}{-2}$

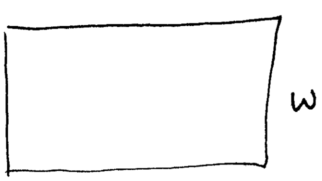
$\sqrt{(x+3)^2} = \sqrt{5}$

$x + 3 = \pm\sqrt{5}$

$-3 \quad -3$

$x = \pm\sqrt{5} - 3$

19. A picture has a length that is 4/3 times larger than its width. It is going to be enlarged (at Walmart) to have an area of 192 square inches. What are the dimensions of the enlarged photo?



$L = \frac{4}{3}w$

$A = L \cdot w$

$192 = \frac{4}{3}w \cdot w$

$3(192) = \left(\frac{4}{3}w^2\right)3$

$\frac{576}{4} = \frac{4w^2}{4}$

$\sqrt{144} = \sqrt{w^2}$

$\pm 12 = w$

$12 = w$

$L = \frac{4}{3}(12)4$

$L = 16$

Length = 16"
Width = 12"

Chapter 5 Material (comp. 3)

1. Name the polynomial based on its degree and number of terms: $3x^5 - 2$

Quintic binomial

2. Give the zeros of the following function. State if any zeros have a multiplicity greater than 1.

$$f(x) = x(x-3)^2(x+4)^3$$

$x = 0$ $x = 3$ (mult. of 2) $x = -4$ (mult. of 3)

3. Classify the polynomial $2x(x-3)(x+2)$ by the number of terms it has and its degree.

$2x(x^2+2x-3x-6)$

$2x(x^2-x-6)$

$2x^3-2x^2-12x$

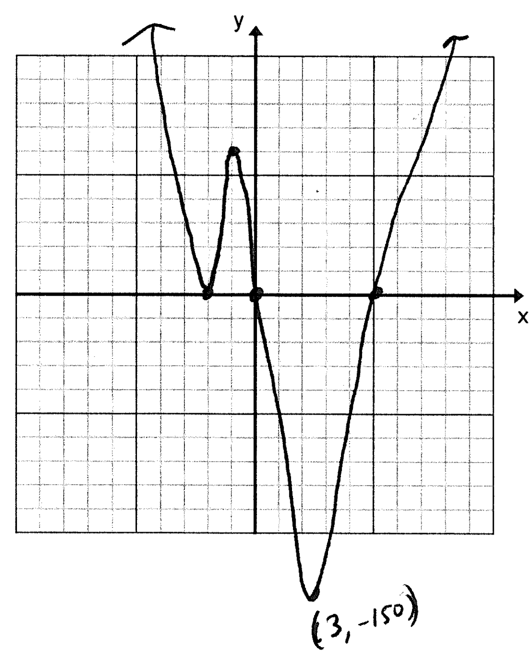
Cubic Trinomial

4. Sketch a possible graph of the polynomial $f(x) = x(x-5)(x+2)^2$ using your knowledge of end behavior of the graph and the zeroes of the polynomial.

zeros: $x=0$, $x=5$, $x=-2$ (double root)

Positive Quartic

X	f(x)
-1	6
3	-150



5. What is the end-behavior of the function $g(x) = -2x^7 + 5x - 2$?

$x \to -\infty,\ g(x) \to \infty$

$x \to \infty,\ g(x) \to -\infty$

6. Factor the polynomial $9x^3 + 6x^2 - 3x$ completely (write it in terms of its linear factors).

$GCF = 3x$

$3x(3x^2 + 2x - 1)$

$3x(x+1)(3x-1)$

7. Find the functions *roots* by factoring. $y = x^4 - 8x^3 + 16x^2$

$GCF = x^2$

$0 = x^2(x^2 - 8x + 16)$

$0 = x^2(x-4)(x-4)$

$0 = x^2(x-4)^2$

$\sqrt{0} = \sqrt{x^2}$

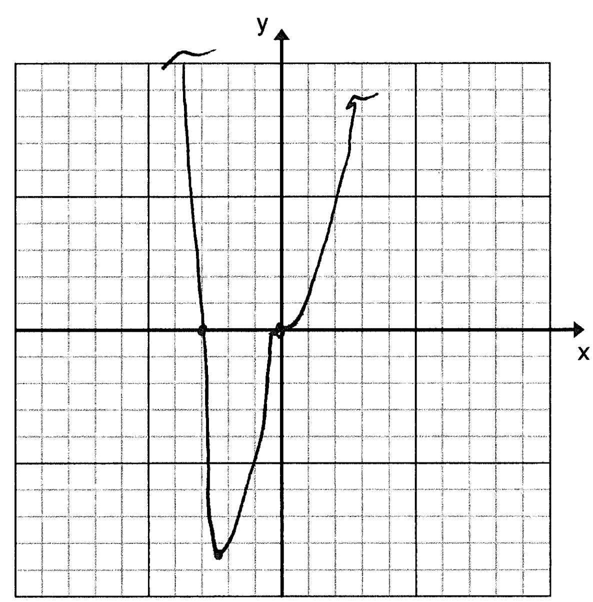
$0 = x$

$0 = x - 4$

$+4 \quad +4$

$4 = x$

8. a. Sketch a graph of the function $y = x^4 + 3x^3$ using your calculator.



$(-2.25, -8.543)$

b. What is the *end behavior* of this function? (1 point)

$x \to -\infty, \ y \to \infty$

$x \to \infty, \ y \to \infty$

c. List the turning point(s) (1 point).

$(-2.25, -8.543)$

d. List the interval(s) of increasing/decreasing (1 point).

Dec: $x < -2.25$

Inc: $x > -2.25$