Name_____ Date_____

Final Exam Review Part 2: Exponential and Logarithmic Functions (Chapter 7)

Without graphing, determine whether each equation represents exponential growth or decay and explain why. $y = a\,b^x$

1. $y = .002(1.05)^x$

Growth → $b > 1$

2. $f(x) = 250\left(\frac{4}{5}\right)^{x+3}$

Decay → $b < 1$

Write each equation in logarithmic form.

3. $6^4 = 1296$

$\log_6 1296 = 4$

4. $5^{-4} = 0.0016$

$\log_5 0.0016 = -4$

Evaluate each expression without a calculator. Show all work.

5. $\log_4 16 = x$

$4^x = 16$

$x = 2$

6. $\log_9 3 = x$

$(9)^x = 3$

$(3^2)^x = 3^1$

$2x = 1$

$x = \frac{1}{2}$

7. $\log_{64} \frac{1}{16} = x$

$64^x = \frac{1}{16}$

$(64)^x = 16^{-1}$

$(4^3)^x = (4^2)^{-1}$

$3x = -2$

$x = -\frac{2}{3}$

8. How much time must you leave an initial investment of \$1500 in an account to grow to \$3200 at a rate of 4.5% compounded continuously? (Give the years and months)

$y = Pe^{rt}$

$\frac{3200}{1500} = \frac{1500e^{.045t}}{1500}$

$\log \frac{32}{15} = \log_e .045t$

$\frac{\log\left(\frac{32}{15}\right)}{.045 \log e} = \frac{.045t \log e}{.045 \log e}$

$16.83746 = t$

$.83746(12) = 10.05$

16 years, 10 months

$y = ab^x \longrightarrow y = a(1 \pm r)^x$

9. The radioactive element Batium decays at a rate 3.75% per year. If you started with 1000 mg how many mgs will be left after 20 years?

$y = 1000(1 - .0375)^{20}$

$y = 465.60$ mg

10. **Write an exponential function of ~~the form $y = ab^x$~~ that has a graph through the given points.**

(1,4), (2,108)

$4 = ab^1$

$\frac{4}{b} = a$

$108 = \left(\frac{4}{b}\right)b^2$

$108 = \frac{4b^2}{b}$

$\frac{108}{4} = \frac{4b}{4}$

$27 = b$

$\frac{4}{27} = a$

So... $y = \frac{4}{27}(27)^x$

11. You put $2234 into a bank account earning 3% annual interest. Find the amount after 12 years in the account if the interest is compounded monthly.

$y = a\left(1 + \frac{r}{n}\right)^{nt}$

$y = 2234\left(1 + \frac{.03}{12}\right)^{12(12)}$

$y = \$3200.62$

Write each equation in exponential form.

12. $\log_5 125 = 3$

$5^3 = 125$

13. $\log_t b = s$

$t^s = b$

14. The half-life of a certain radioactive material is half an hour. If you start with 250 mg of the material, how much is left in 24 hours?

$y = a\left(\frac{1}{2}\right)^{\frac{t}{h}}$

$y = 250\left(\frac{1}{2}\right)^{\frac{24}{1/2}}$

$y = 8.88 \times 10^{-13}$

Notice on the calculator you get the answer 8.88... E -13, which means 8.88×10^{-13}

→ Right 1, down 2

15. Graph $y = -2(2)^{x-1} - 2$ and list the domain, range and asymptote.

PF = 2^x

x	2^x	$-2(2^x)$
-3	1/8	-1/4
-2	1/4	-1/2
-1	1/2	-1
0	1	-2
1	2	-4
2	4	-8
3	8	-16

Domain:

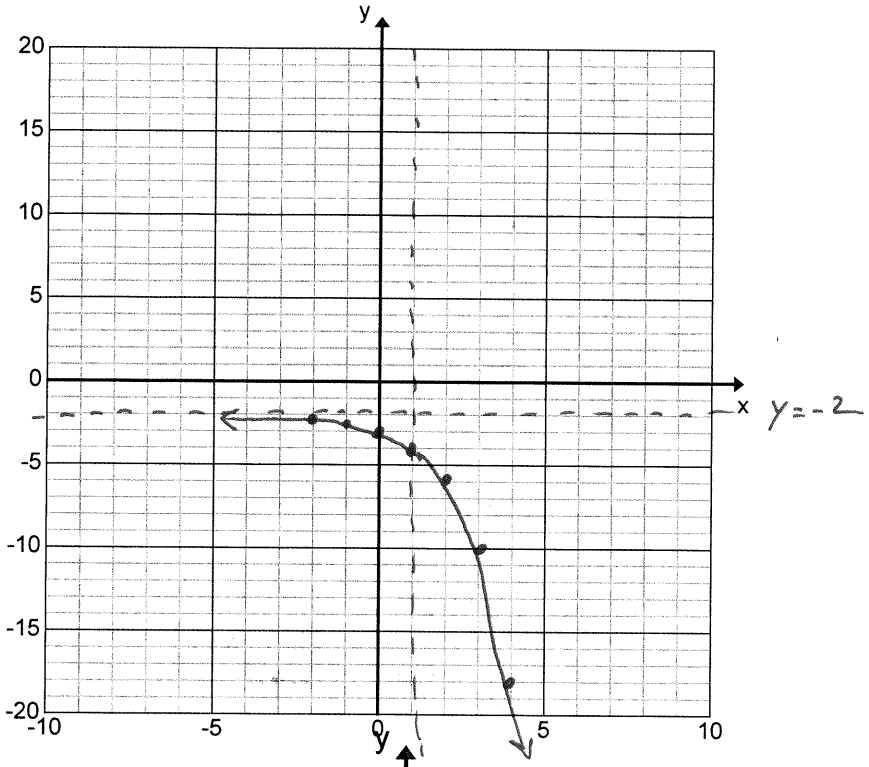
R

Range:

$y < -2$

Asymptote:

$y = -2$



16. Graph the function $y = \log_3(x+2)+1$.

→ left 2, up 1

a) What's the log parent function?

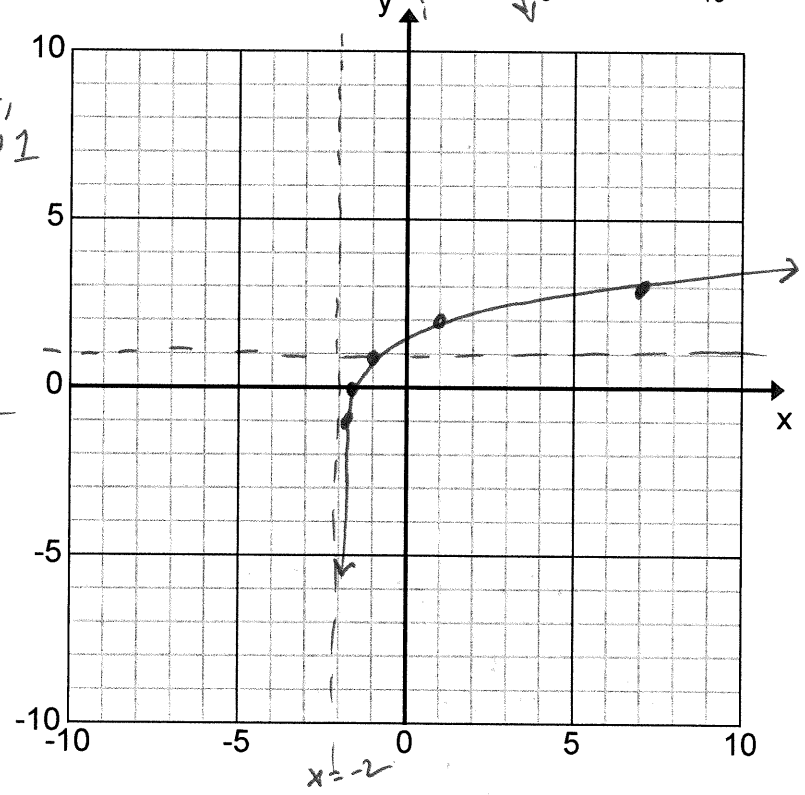
$\log_3 x$

b) Now, find the *inverse* parent function.

3^x (just "throw away" the log from above)

c) Make a chart for the *inverse parent function.*

x	$y = 3^x$
-2	1/9
-1	1/3
0	1
1	3
2	9
3	27



d) Switch the values of your *inverse chart* to graph the logarithm function.

x	$y = \log_3 x$
1/9	-2
1/3	-1
1	0
3	1
9	2
27	3

e) Domain, range and asymptote:

$x > -2$ | R | $x = -2$

base, value, power

Solve the following equations. Round any answers to the nearest hundredth.

17. $\ln 3x = 8$

$\frac{e^8}{3} = \frac{3x}{3}$

$993.65 = x$

18. $\frac{1}{2}\ln 5x = (4)^2$

$\ln 5x = 8$

$\frac{e^8}{5} = \frac{5x}{5}$

$596.19 = x$

19. $2\log 3x - 4 = 2$

$+4 \quad +4$

$\frac{2\log 3x}{2} = \frac{6}{2}$

$\log 3x = 3$

$\frac{10^3}{3} = \frac{3x}{3}$ $\quad 333.33 = x$

Write each expression as a single logarithm. Simplify as much as possible.

20. $\log_5 3 + \log_5 6$

$\log_5 (3 \cdot 6)$

$\log_5 18$

21. $\log_2 32 - \log_2 8$

$\log_2 \left(\frac{32}{8}\right)$

$\log_2 (4) = 2$

(since $2^x = 4$, and $x = 2$)

22. $\frac{1}{2}\log x + \frac{1}{3}\log y - 2\log z$

$\log x^{\frac{1}{2}} + \log y^{\frac{1}{3}} - \log z^2$

$\log \left(x^{\frac{1}{2}} y^{\frac{1}{3}}\right) - \log z^2$

$\log \left(\frac{x^{\frac{1}{2}} y^{\frac{1}{3}}}{z^2}\right)$

Solve each equation. Give both the exact answer *and* the rounded answer to the nearest hundredth.

23. $4^x = 16$

$x = 2$

24. $9^{y-3} = 81$

$(3^2)^{y-3} = 3^4$

$3^{2y-6} = 3^4$

$2y - 6 = 4$

$+6 \quad +6$

$\frac{2y}{2} = \frac{10}{2}$

$y = 5$

Expand each logarithm.

25. $\log_5 \frac{a}{b}$

$\log_5 a - \log_5 b$

26. $\log_3 \left(x \cdot y^{\frac{1}{2}} \right)$

$\log_3 x + \log_3 y^{\frac{1}{2}}$

$\log_3 x + \frac{1}{2} \log_3 y$

27. $\log \frac{2x^2 y}{3k^3} = \log 2x^2 y - (\log 3k^3)$

$= \log 2 + \log x^2 + \log y - (\log 3 + \log k^3)$

$= \log 2 + 2\log x + \log y - \log 3 - 3\log k$

Use logarithm properties to *evaluate* the following expressions.

28. $\log_2 160 - \log_2 5$

$= \log_2 \left(\frac{160}{5} \right)$

$= \log_2 32$, so $2^x = 32$

$x = 5$

29. $\log_7 14 - \log_7 2$

$= \log_7 \frac{14}{2}$

$= \log_7 7$, so $7^x = 7$

$x = 1$

30. Suppose an initial investment of *500* dollars is put into an account that pays 4.5% compounded quarterly. About how long would it take to double your initial investment? Give the number of years and months.

$$\frac{1000}{500} = \frac{500\left(1 + \frac{.045}{4}\right)^{4t}}{500}$$

$$\log 2 = \log\left(1 + \frac{.045}{4}\right)^{4t}$$

$$\frac{\log 2}{4\log(1.01125)} = \frac{4t\log(1.01125)}{4\log(1.01125)}$$

$$15.49 = t$$

↳ 15 years and about 6 months