Chapter 2 Part 2 Notes: **Rational Functions**

Graphing Rational Functions

-Definition: A *rational function* has the form of $y = \frac{f(x)}{g(x)}$, where $f(x)$ *and* $g(x)$ are both polynomials and $g(x) \neq 0$.

-Steps for graphing a rational function:

1) Locate the zeros (x-intercepts) of $f(x)$. (Just set the numerator equal to 0 and solve). A zero of f (x) will be an x intercept of $y = \frac{f(x)}{g(x)}$ unless it is also a zero of $g(x)$. If this is the case then there is a *hole* at that domain value.

2) Find the y-intercept by substituting *0* for your independent variable.

3) Find where $g(x)$ equals zero. These are your *vertical asymptotes* (unless of course a factor of the denominator cancels with a factor of the numerator. In this case, you have a *hole* rather than a *vertical asymptote* at that domain value).

4) Find the horizontal asymptote:

a) If the degree of $f(x)$ < degree of $g(x)$, horizontal asymptote @ y= 0 (x –axis).

b) If degree of $f(x)$ = degree of $g(x)$, horizontal asymptote @ $y = \frac{LC\ of\ f(x)}{LC\ of\ g(x)}$

c) If degree of $f(x)$ > degree of $g(x)$, no horizontal asymptote. You must then test for a slant or oblique asymptote by using division. The quotient of the two functions will be your asymptote.

***Note**: It is possible for a function to cross horizontal, oblique, and slant asymptotes. So you need to determine if that happens as well.

5) Test for symmetry (even or odd).

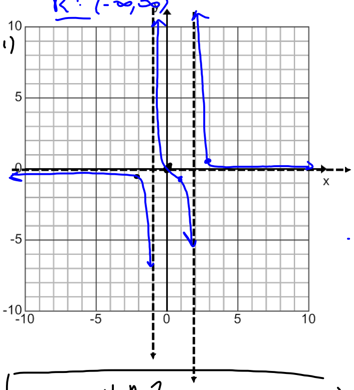
6) Test each interval of your graph to see where your function will be. Use limits to determine what your function approaches as x goes to negative and positive infinity.

Analyze the following rational functions. Then, graph.

D: $(-\infty,-1)\cup(-1,2)\cup(2,\infty)$

R: $(-\infty,\infty)$

$g(x)=\frac{x}{x^2-x-2} = \frac{x}{(x-2)(x+1)}$



Zeroes: $0=\frac{x}{x^2-x-2}$

$0=x$

(0,0)

y-int: $g(0)=\frac{0}{0^2-0-2}=0$

(0,0)

V.A.: $x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2 \quad x=-1$

H.A.: @ $g(x)=0$

Cross H.A.? $0=\frac{x}{x^2-x-2}$ yes @ (0,0)

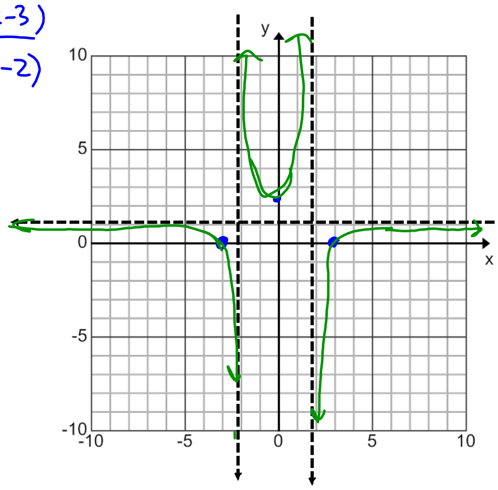
$0=x$

Test points: $g(-2)=\frac{-2}{(-2)^2-(-2)-2}=\frac{-2}{6}=-\frac{1}{3}$

$g(1)=\frac{1}{1^2-1-2}=\frac{1}{-2}$

$g(3)=\frac{3}{3^2-3-2}=\frac{3}{9-3-2}=\frac{3}{4}$

$f(x)=\frac{x^2-9}{x^2-4} = \frac{(x+3)(x-3)}{(x+2)(x-2)}$



Zeros: $0=(x+3)(x-3)$

(-3,0) (3,0)

Y-int: $f(0)=\frac{0^2-9}{0^2-4}=\frac{9}{4}$

$(0,\frac{9}{4})$

VA: $x=\pm 2$

HA: $\frac{\text{LC Num}}{\text{LC den}}=\frac{1}{1}=1$

$f(x)=1$

Cross HA? $(x^2-4)(1)=\left(\frac{x^2-9}{x^2-4}\right)(x^2-4)$

$x^2-4=x^2-9$

$-4=-9$

No

even? $f(-x)=\frac{(-x)^2-9}{(-x^2)-4}$

$f(-x)=\frac{x^2-9}{x^2-4}$ ✓

$$g(x) = \frac{x^2 - x - 2}{x - 1} = \frac{(x-2)(x+1)}{x-1}$$

Zeros: $0 = (x-2)(x+1)$
$(2,0), (-1,0)$

y-int: $g(0) = 2$
$(0,2)$

VA @ $x = 1$

No H.A. but there is a slant:

$$\begin{array}{c|ccc} 1 & 1 & -1 & -2 \\ & & 1 & 0 \\ \hline & 1 & 0 & -2 \end{array}$$

$x + \frac{-2}{x-1}$

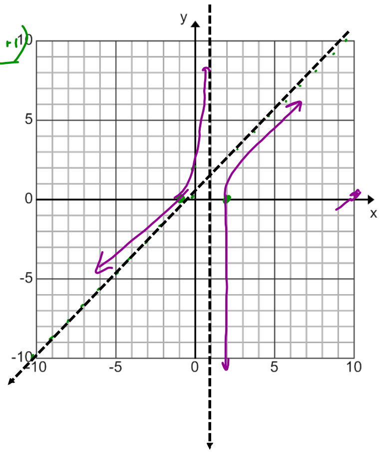
↑ slant asymptote

Cross slant?

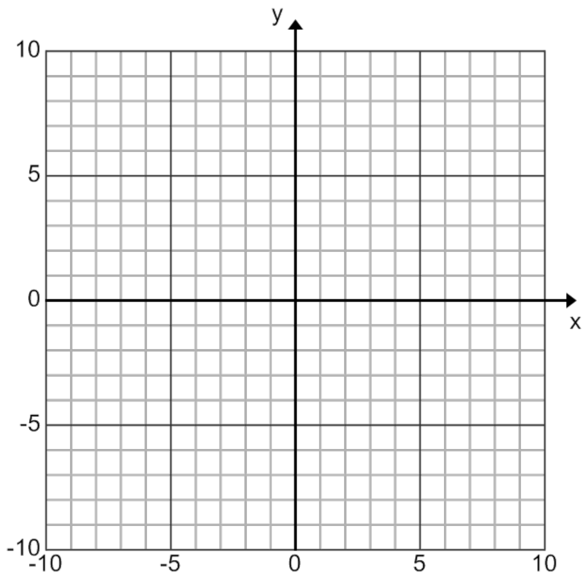
$(x-1)(x) = \left(\frac{x^2 - x - 2}{x-1}\right)(x-1)$

$x^2 - x = x^2 - x - 2$

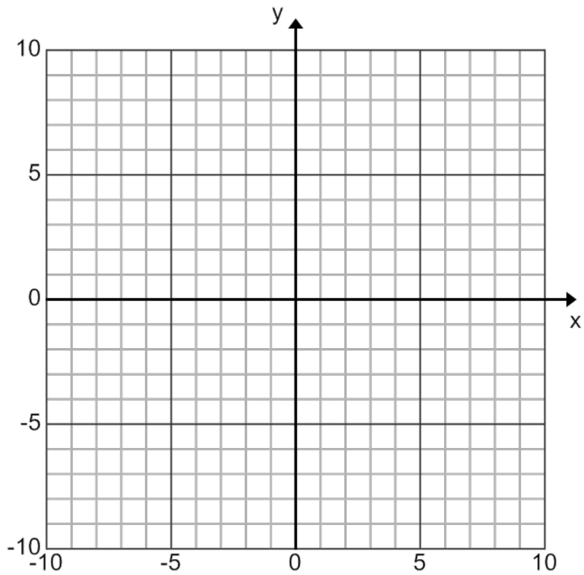
$0 = -2$ NO



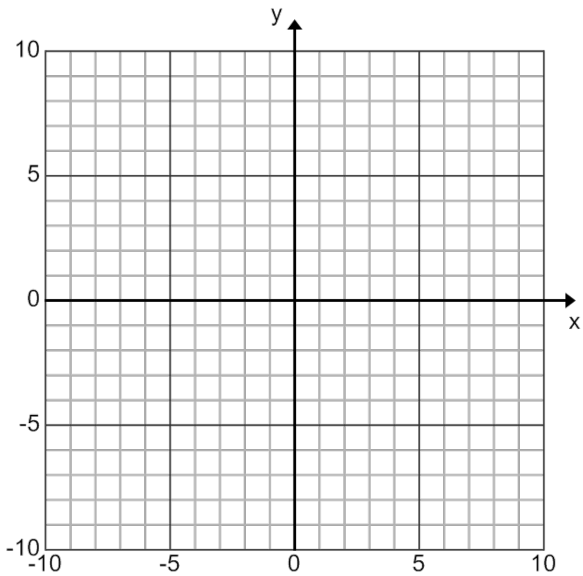
$$g(x) = \frac{2x^2 + 10x + 12}{x^2 - 9}$$



$$f(x)=\frac{3x^3+x-5}{x^2+1}$$



$$g(x)=\frac{4x^3-8x^2-5x}{x-1}$$

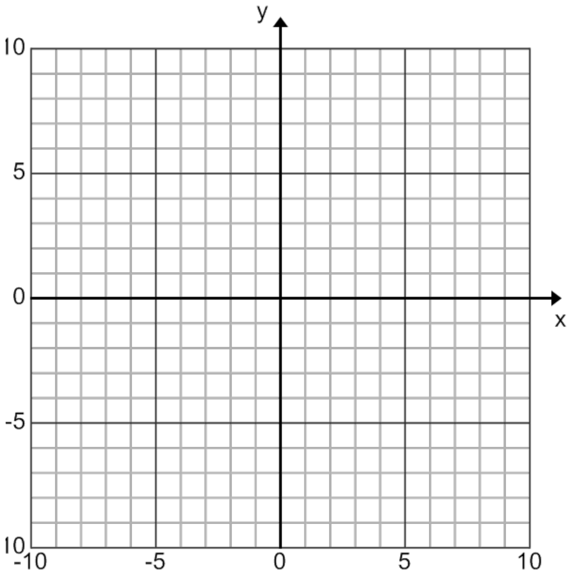


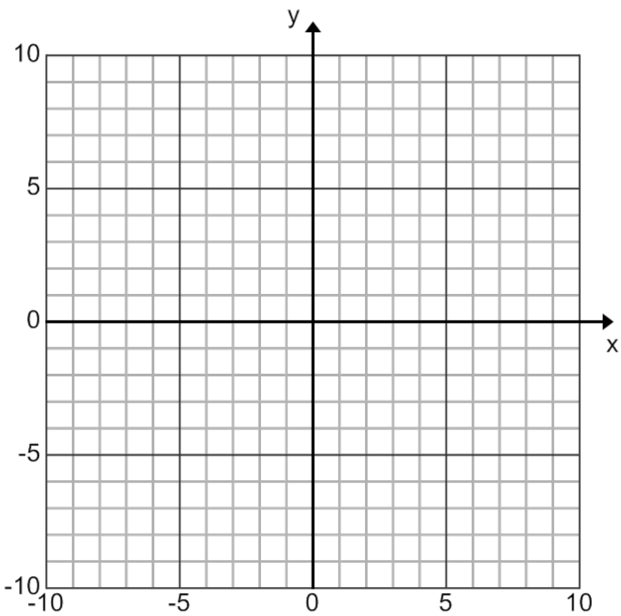
Graph the following rational functions.

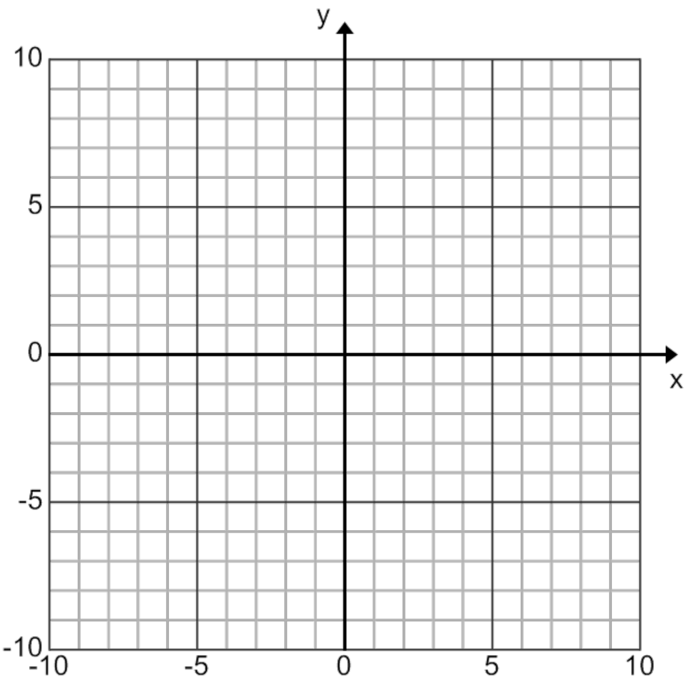
22. $y = \frac{x - 2}{(x + 2)(x - 2)}$ **23.** $y = -\frac{x}{x(x - 1)}$ **24.** $y = \frac{5 - x}{x^2 - 1}$

25. $y = \frac{x^2 - 2}{x + 2}$ **26.** $y = \frac{x^2 - 4}{x^2 + 4}$ **27.** $y = \frac{x + 3}{x^2 - 9}$

28. $y = \frac{x^2 - 25}{x - 4}$ **29.** $y = \frac{(x - 2)(2x + 3)}{(5x + 4)(x - 3)}$ **30.** $y = \frac{15x^2 - 7x - 2}{x^2 - 4}$



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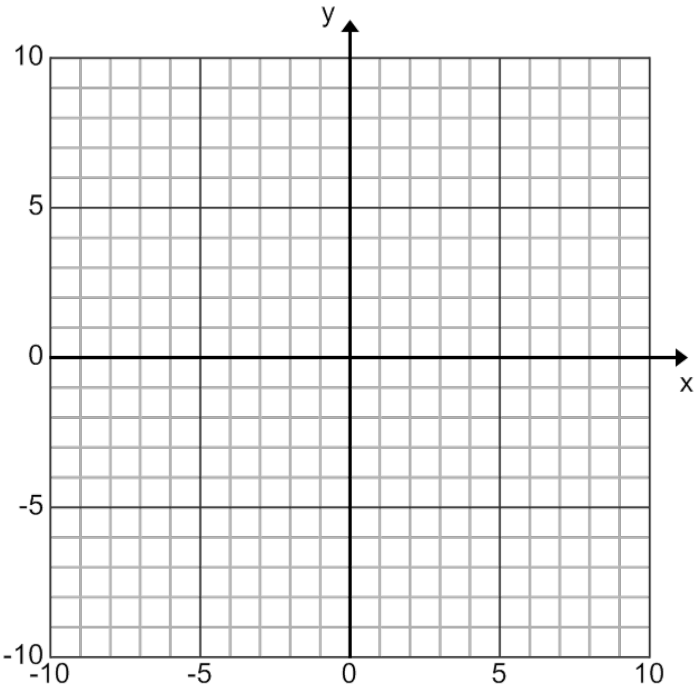
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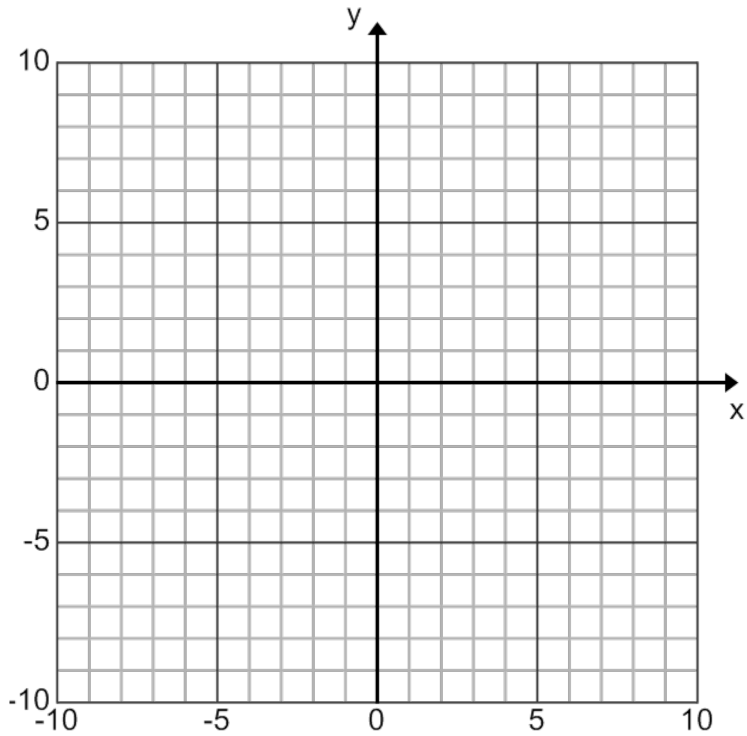
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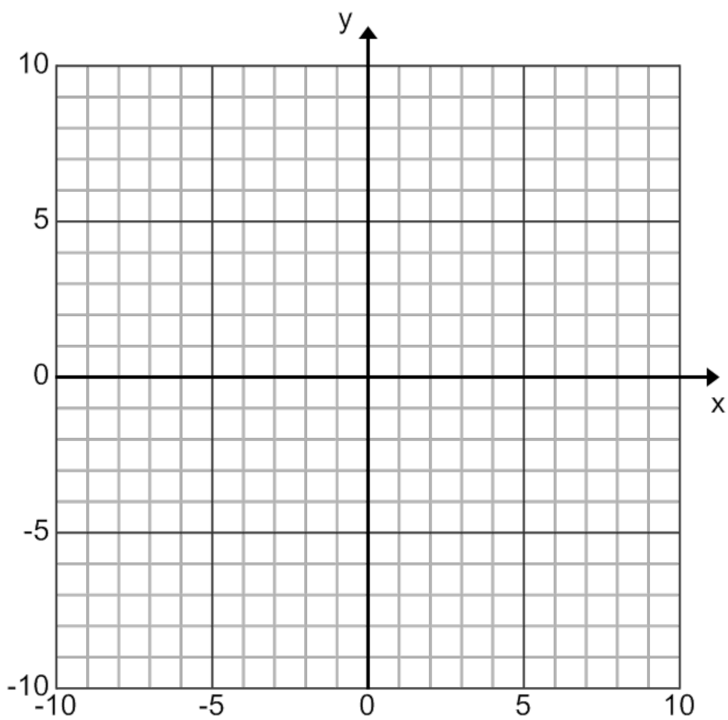
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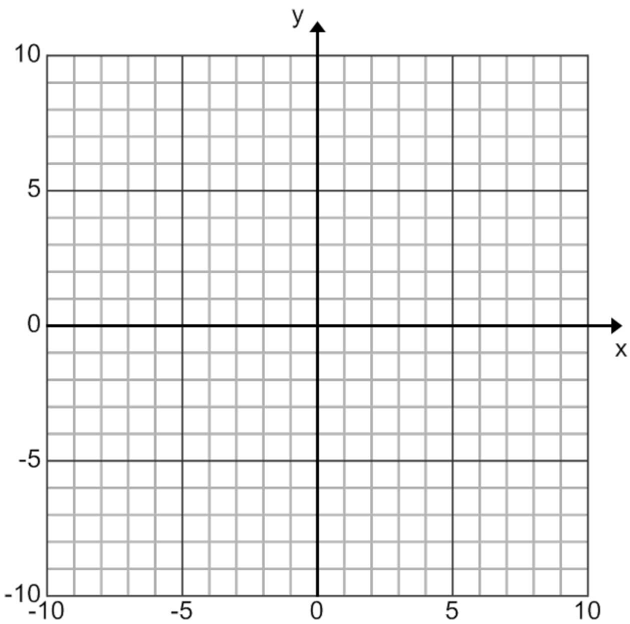
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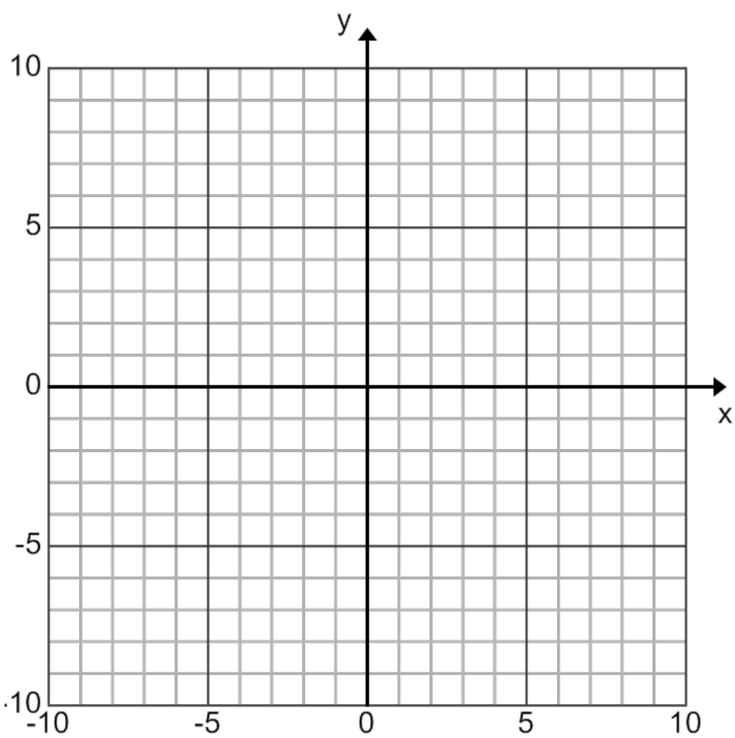
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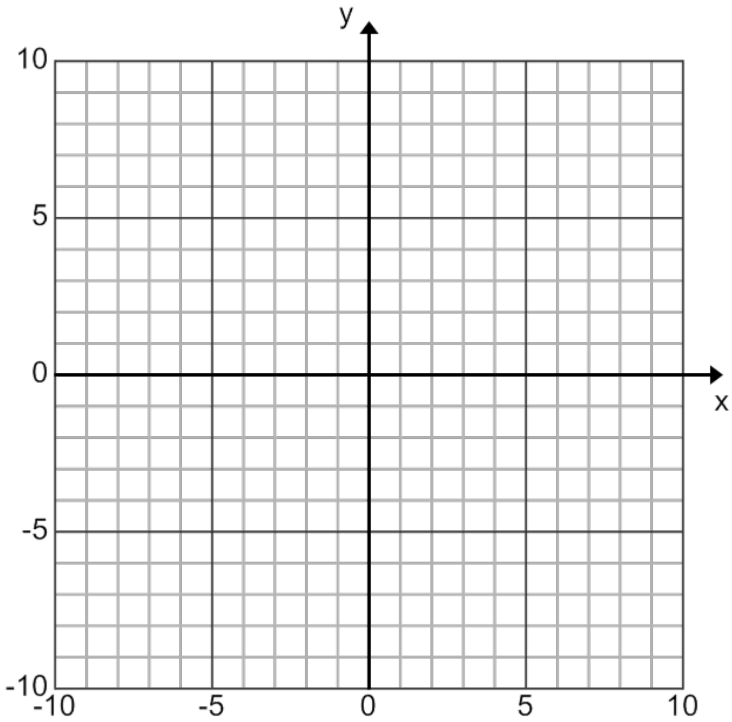
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