Name Key Date

PreCalc H -1 – Final Review

v2

Answer each question completely. Follow all directions. Show all work.
What does it mean if a function is even? Odd? Neither?

Even = Symmetrical about the y-axis → If (a, b) is on graph $(-a, b)$ is too

Odd = " " the origin → If (a, b) is on graph, $(-a, -b)$ is too

How do you determine if a function is even, odd, or neither?

Even: $f(-x) = f(x)$

Odd: $-f(-x) = f(x)$

Determine if the following functions are even, odd, or neither.

$f(x) = x^2 - 5$

$f(-x) = (-x)^2 - 5$

$f(-x) = x^2 - 5$ → even

b) $g(x) = -x^3 - 9x$

$-g(-x) = -(-x)^3 - 9\cancel{f}(-x)$

$-g(-x) = -(-(x)^3) + 9x$

$-g(-x) = x^3 + 9x$

$g(-x) = -x^3 - 9x$ → odd

c) $f(x) = \frac{x^2 + 5}{x}$

$f(-x) = \frac{(-x)^2 + 5}{-x} = \frac{x^2 + 5}{-x} = \frac{-x^2 - 5}{x}$

$-f(-x) = \frac{(-x)^2 + 5}{-x}$

$\frac{f(-x)}{-1} = \frac{\frac{x^2 + 5}{-x}}{-1}$

$f(-x) = x^2 + 5$ → odd

d) $g(x) = 2x^{2/3}$

$g(-x) = 2\sqrt[3]{(-x)^2}$

$g(-x) = 2\sqrt[3]{x^2}$

$g(-x) = 2x^{\frac{2}{3}}$ → even

Given the function $f(x)=\frac{x^5}{x^{1/3}}$, $g(x)=\frac{1}{x^6}$, please do the following:

Find $(f\cdot g)(x)$. $= \frac{x^5}{x^{\frac{1}{3}}}\cdot\frac{1}{x^6} = \boxed{\frac{1}{x^{\frac{4}{3}}}} = \frac{1}{x\sqrt[3]{x}}\cdot\frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} = \boxed{\frac{\sqrt[3]{x^2}}{x^2}}$

Is $(f\cdot g)(x)$ even, odd, or neither? Why?

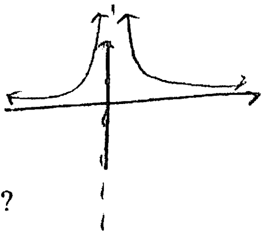
$(f\cdot g)(-x) =$

$\frac{1}{\sqrt[3]{(-x)^4}} = \frac{1}{\sqrt[3]{x^4}} = \frac{1}{x^{4/3}}$ → even

What is the domain and range of $(f\cdot g)(x)$?

Domain $(-\infty,0)\cup(0,\infty)$

Range $(0,\infty)$



On what interval(s) does this function increase? Decrease?

Increase: $(-\infty,0)$

Decrease: $(0,\infty)$

Is there a relative min/max? If so where? If not why not?

Neither

A

44 Which of the following *most* accurately describes the translation of the graph $y=(x+3)^2-2$ to the graph of $y=(x-2)^2+2$?

Right 5, up 4

[✓ A] up 4 and 5 to the right

B down 2 and 2 to the right

C down 2 and 3 to the left

D up 4 and 2 to the left

15 What is the cusp of the graph of the function below?

$f(x) = -|x + 1| + 3$

$(-1, 3)$

Ⓐ (1, 3)

[✓ Ⓑ (–1, 3)]

Ⓒ (–1, –3)

Ⓓ (1, –3)

If $N(x) = f(g(h(x))) = \left(\frac{x^2 - 6x + 5}{(x-2)}\right)^5$, find $f(x)$, $g(x)$, and $h(x)$.

~~$x^2 - 4x + 4$~~

$$\boxed{\begin{aligned} g(x) &: \frac{x^2 - 2x - 3}{x} \\ h(x) &\quad x - 2 \\ f(x) &: x^5 \end{aligned}}$$

Check:

$$g(h(x)) = \frac{(x-2)^2 - 2(x-2) - 3}{x-2}$$

$$= \frac{x^2 - 4x + 4 - 2x + 4 - 3}{x-2}$$

$$= \frac{x^2 - 6x + 5}{x-2}$$

$$f(g(h(x)) = \left(\frac{x^2 - 6x + 5}{x-2}\right)^5$$

Let $f(x) = \sqrt{2x+8}$, $g(x) = \frac{1}{x^2 - 4}$

Determine $g(f(x))$. List domain.

$$g(f(x)) = \frac{1}{(\sqrt{2x+8})^2 - 4} = \frac{1}{2x + 8 - 4} = \boxed{\frac{1}{2x+4}}$$

Domain:

$2x + 8 \geq 0$

$2x \geq -8$

$x \geq -4$

$2x + 4 \neq 0$

$2x \neq -4$

$x \neq -2$

$\boxed{[-4, -2) \cup (-2, \infty)}$

If a function is one-to-one, then each output [redacted] can only have one input [redacted]

$y=x^2$

$(-2,4)$  $(2,4)$ → not one-to-one

If $f(x)$ and $g(x)$ are inverses of each other and $(a,\ b)$ is a point on the graph of $f(x)$, then the point

(b, a) will be on the graph of $g(x)$.

If $f(x)$ and $g(x)$ are inverses of each other, then the domain of $f(x)$ is the range $g(x)$, and the domain of $g(x)$ is the range of $f(x)$.

Find the inverse of the function $f(x) = -\dfrac{5x+1}{x-1}$. (Careful about that negative sign in front). Then, find the domain and range of the inverse function.

$$x = \frac{-(5y+1)}{y-1} \rightarrow -x = \frac{5y+1}{y-1} \longrightarrow -x(y-1) = 5y+1$$

$$-xy + x = 5y+1$$

$$x - 1 = xy + 5y$$

$$x - 1 = y(x+5)$$

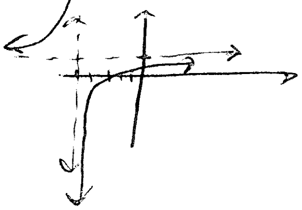
$$\frac{x-1}{x+5} = y$$

$$\boxed{f^{-1}(x) = \frac{x-1}{x+5}}$$

Domain:
$(-\infty,-5) \cup (-5,\infty)$
Range: $(-\infty,1) \cup (1,\infty)$

$$x+5 \overline{\smash{)}\, x-1}^{\,1} \quad -(x+5) \quad -6$$

$\rightarrow$ so $f^{-1}(x) = 1 - \dfrac{6}{x+5}$



Find $\dfrac{g(t+x)-g(t)}{x}$, when $g(t) = 6 - 3t + t^2$.

$$\frac{6 - 3(t+x) + (t+x)^2 - (6 - 3t + t^2)}{x}$$

$$= \frac{\cancel{6} \cancel{-3t} - 3x \cancel{+t^2} + 2tx + x^2 \cancel{-6} \cancel{+3t} \cancel{-t^2}}{x}$$

$$= \frac{-3x + 2tx + x^2}{x} = \boxed{-3 + 2t + x}$$

$$\text{Solve: } \overset{(x-3)}{\frac{(x-2)}{x+5}}+\overset{(x+5)}{\frac{(x+1)}{x-3}}=\frac{5x-9}{x^2+2x-15}$$

$$\frac{x^2-5x+6+x^2+6x+5}{(x+5)(x-3)} = \frac{5x-9}{x^2+2x-15}$$

[redacted]

$$2x^2+x+11 = 5x-9$$

[redacted]

[redacted]

$$2x^2-4x+20=0$$

$$2(x^2-2x+10)=0$$

$$x=\frac{2\pm\sqrt{4-4(1)(10)}}{2}$$

$$x=\frac{2\pm\sqrt{4-40}}{2}$$

$$x=\frac{2\pm\sqrt{-36}}{2}=\frac{2\pm 6i}{2}=\boxed{1\pm 3i}$$

No intersection

thus, no real solution

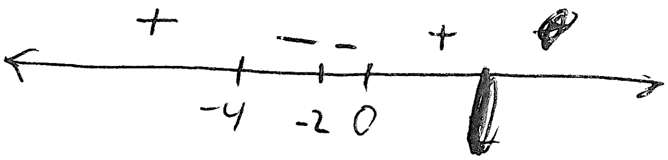
Solve the inequality $\frac{n^2+4n+4}{n^2+4n}>0$.

X-ints: $n^2+4n+4=(n+2)(n+2)$

$n=^-2$ ← double

Vertical asymptotes: $n(n+4)$

$n=0 \quad n=-4$



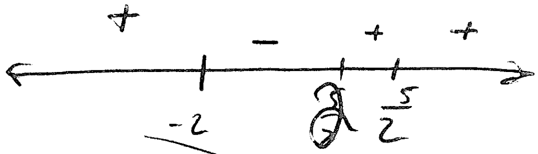
$(-\infty,-4)\cup(0,\infty)$

Solve the inequality $\frac{|2x-5|}{x^2-4}<0$.

X-int : $2x = 5$

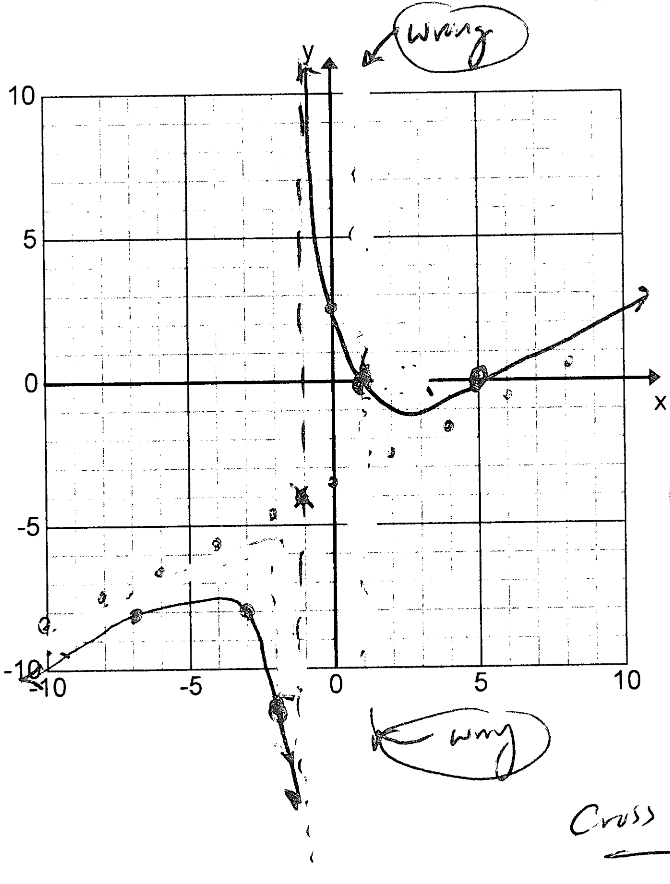
$x = \frac{5}{2}$

Asymptotes : $x = \pm 2$



$$(-2, 2)$$

Analyze the rational function $g(x)=\frac{x^2-6x+5}{2x+2}$ and also graph.



X-ints : $(x-1)(x-5)=0$

$x=1 \quad x=5$

Y-int : $(0, 5/2)$

VA @ $x=-1$

No HA :

Go slant

$$\frac{1}{2}x - \frac{7}{2} + \frac{12}{2(x+1)} = \frac{6}{x+1}$$

$$2x+2 \overline{) x^2-6x+5}$$

$-(x^2+1x)$

$-7x+5$

$-(-7x-7)$

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Cross slant?

$$(2x+2)\left(\frac{1}{2}x-\frac{7}{2}\right) = \frac{x^2-6x+5}{\cancel{(2x+2)}}\cancel{(2x+2)}$$

$$x^2-7x+x-7 = x^2-6x+5$$

$-7 \neq 5 \rightarrow$ no

$0 = \frac{-3}{4}x + \frac{25}{4}$

Two sides of a rectangle lie on along the x and y axes. One vertex of the rectangle lies along the line $3x+4y=25$. $\rightarrow 4y = -3x + 25 \rightarrow \boxed{y = \frac{-3}{4}x + \frac{25}{4}}$

$\frac{-25}{4} = \frac{-3}{4}x$

$\left(\frac{-25}{4}\right)\left(\frac{-4}{3}\right) = x$

$\frac{25}{3} = x$

Express the area (A) of the rectangle as a function of the x-coordinate A(x).

What is the domain of the Area function?

Domain : → common sense $0 < x < \frac{25}{3}$

↳ $(0, \frac{25}{3})$

$y = \frac{-3}{4}(0) + \frac{25}{4}$

$= \frac{25}{4} = y$ ↙ y-int

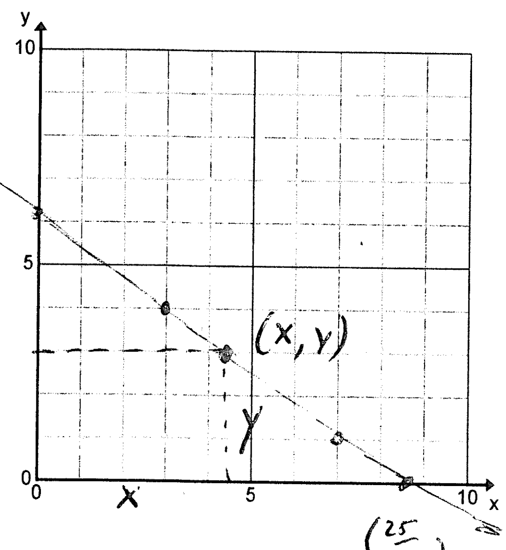
What dimensions of the box would yield the maximum area of the rectangular region?

Remember, $y = \frac{-3}{4}x + \frac{25}{4}$

Area = x · y

$A(x) = x\left(-\frac{3}{4}x + \frac{25}{4}\right)$

$$A(x) = -\frac{3}{4}x^2 + \frac{25}{4}x$$



(3, 4)

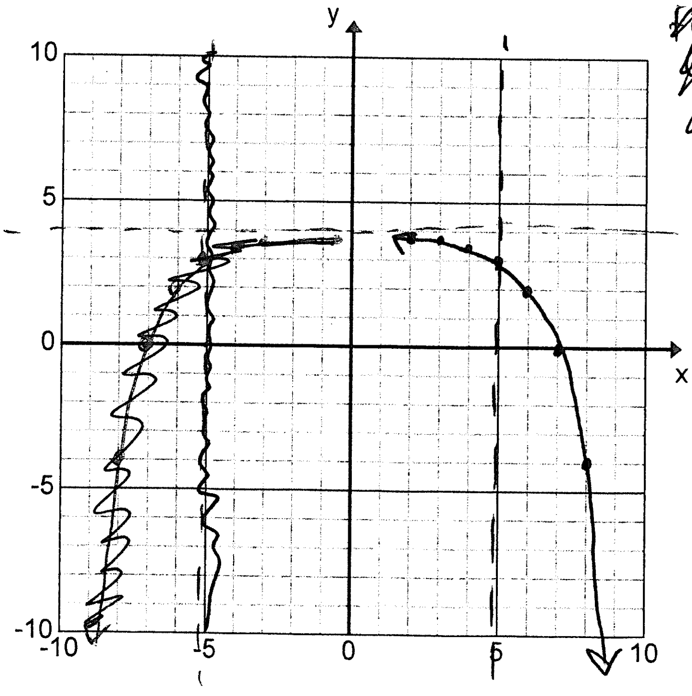
Max :

$$x = \frac{\frac{-25}{4}}{2\left(\frac{-3}{4}\right)} = \frac{\frac{-25}{4}}{-\frac{3}{2}} = \frac{-25}{4}\cdot\left(\frac{-2}{3}\right) = \frac{25}{6}$$

$$y = \frac{-3}{4}\left(\frac{25}{6}\right) + \frac{25}{4} = \frac{-25}{8} + \frac{25}{4} = \frac{25}{8}$$

$\boxed{x = \frac{25}{6} \quad y = \frac{25}{8}}$

Graph the function $f(x) = -2^{(+x-5)} + 4$ using your knowledge of transformations.



[illegible]

x	$y = 2^x$	$y = -2^x$
-3	1/8	-1/8
-2	1/4	-1/4
-1	1/2	-1/2
0	1	-1
1	2	-2
2	4	-4
3	8	-8

Simplify $\sqrt{\frac{25^3 \cdot 5^n}{125^{2-n}}}$ using common base

$$\sqrt{\frac{(5^2)^3 \cdot 5^n}{(5^3)^{(2-n)}}} = \sqrt{\frac{5^6 \cdot 5^n}{5^{6-3n}}} = \sqrt{\frac{5^{6+n}}{5^{6-3n}}}$$

$$= \sqrt{5^{4n}} = \boxed{5^{2n}}$$

Solve the equation $e^x = 4^{2x-1}$.

$$\ln e^x = \ln 4^{(2x-1)}$$

$$x = (2x-1)\ln 4$$

$$x = (2x)\ln 4 - \ln 4$$

$$x - (2x)\ln 4 = -\ln 4$$

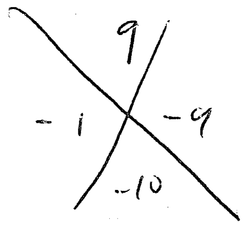
$$x(1 - 2\ln 4) = -\ln 4$$

$$x = \frac{-\ln 4}{1 - 2\ln 4} \approx .7821$$

Solve $3^{2x+1} - 10 \cdot 3^x + 3 = 0$ for x.

$3^1 \cdot 3^{2x} - 10 \cdot 3^x + 3 = 0$

$u = 3^x$



$3u^2 - 10u + 3 = 0$

$3u^2 - 9u - u + 3 = 0$

$3u(u-3) - 1(u-3) = 0$

$(3u-1)(u-3) = 0$

$3 \cdot 3^x - 1 = 0$ $\quad$ $3^x - 3 = 0$

$3^x = \frac{1}{3}$ $\quad$ $3^x = 3$

$\boxed{x = -1}$ $\quad$ $\boxed{x = 1}$

ed

Write $P(x) = 2x^3 - 9x^2 + 3x + 4$, in linear factor form.

x-ints = 4, 1, -½

P $(2x+1)(x-4)(x-1)$

$\frac{p's}{q's} = \pm 4, \pm 2, \pm 1, \pm \frac{1}{2}$

Graph $f(x) = -3x^2 - 6x + 4$. Include as many "critical points" on the graph as you can. Determine the roots.

$x = \frac{6}{-3(2)} = -1$

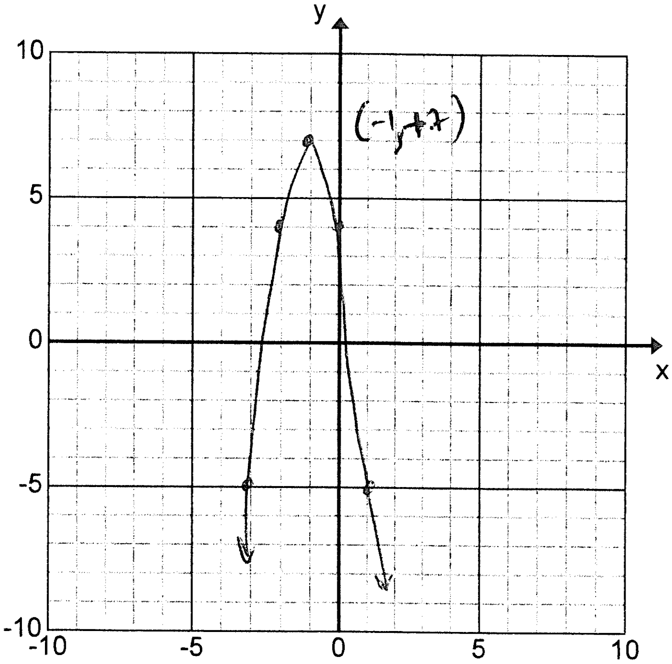
$f(-1) = -3(-1)^2 - 6(-1) + 4$

$= -3(1) + 6 + 4$

$= -3 + 10$

$= 7$

vertex = $(-1, 7)$



$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(-3)(4)}}{2(-3)}$$

$$x = \frac{6 \pm \sqrt{36 + 48}}{-6}$$

$$\frac{6 \pm \sqrt{84}}{-6} = \frac{6 \pm 9.17}{-6} \rightarrow (-2.53, 0) \quad (.53, 0)$$

Simplify as much as possible. $\dfrac{\left[\left(2x^{-5}y\right)^{5}-\left(4y^{25/2}x^{-125/2}\right)^{2/5}\right]^{3}}{5x^{-6}yz}$

$* \sqrt[5]{4^2} = \sqrt[5]{16}$

$4^{\frac{2}{5}} = (2^2)^{\frac{2}{5}} = 2^{\frac{4}{5}}$

$$\frac{\left[2^5 x^{-25} y^5 - (4^{\frac{2}{5}} y^5 x^{-25})\right]^3}{5x^{-6}yz}$$

$$= \frac{\left(2^5 x^{-25} y^5 - 2^{\frac{4}{5}} x^{-25} y^5\right)^3}{5x^{-6}yz}$$

$$= \frac{\left[(2^5 - 2^{\frac{4}{5}})(x^{-25}y^5)\right]^3}{5x^{-6}yz}$$

$$= \frac{(2^5 - 2^{\frac{4}{5}})^3 (x^{-75}y^{15})}{5x^{-6}yz}$$

$$= \frac{(2^5 - 2^{\frac{4}{5}})^3\, y^{14}}{5x^{69}z}$$

you could do this out as, but definitely not as simple

$(2^5 - 2^{4/5})(2^5 - 2^{4/5})(2^5 - 2^{4/5})$