

PreCalc H - 2 - Final - Review

COMP 1

OK Calculator.

Convert 52.23567° into degrees minutes seconds. Round to the nearest second.

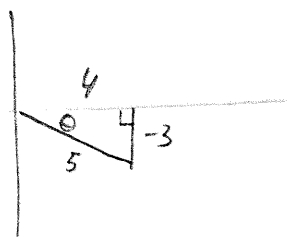
$$52^\circ, 14 \text{ min } 8 \text{ seconds}$$

Convert $234^\circ 52' 10''$ into decimal degrees. Round to the nearest hundredth degree.

$$234^\circ + \frac{52^\circ}{60} + \frac{10^\circ}{3600} = 234.87^\circ$$

No Calculator.

Given that $\sin \theta = -\frac{3}{5}$ and $\frac{3\pi}{2} \leq \theta \leq 2\pi$. Determine all five other trigonometric relations.



$$\begin{aligned} \sin \theta &= -\frac{3}{5} & \csc \theta &= -\frac{5}{3} \\ \cos \theta &= \frac{4}{5} & \sec \theta &= \frac{5}{4} \\ \tan \theta &= -\frac{3}{4} & \cot \theta &= -\frac{4}{3} \end{aligned}$$

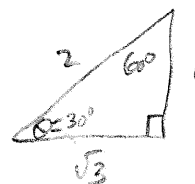
Determine all values of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \theta$

$$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \theta \rightarrow$$

$$\cos \theta = -\frac{\sqrt{3}}{2}$$

$$\theta = 150^\circ \pm 360n$$

$$\theta = 210^\circ \pm 360n$$



$$\theta = \frac{5\pi}{6} \pm 2\pi n$$

$$\theta = \frac{7\pi}{6} \pm 2\pi n$$

$$\{n \in \mathbb{Z}\}$$

OK Calculator.

Fill in the blank with best answer.

The unit circle has a radius of 1 unit and is centered at the origin.

The $-\cot \theta$ measures the slope of a tangent line to the unit circle.

The $\sin \theta$ determines the y-coordinate of a point on the unit circle.

The $\cos \theta$ determines the x-coordinate of a point on the unit circle.

Re-write each angle measure using the other unit, either degrees or radians. SHOW WORK.

1. 200°

$$\frac{200}{180} = \frac{X}{\pi}$$

$$X = 3.49 \text{ radians}$$

2. $\frac{5\pi}{2}$

$$\frac{X}{180^\circ} = \frac{5\pi}{2\pi}$$

$$X = 450^\circ$$

3. 4

$$\frac{X}{180} = \frac{4}{\pi}$$

$$X = 229.18^\circ$$

A sector of a circle has area 15cm^2 and radius 4 cm. Find central angle to the nearest tenth unit.

$$15 = \frac{\theta}{360} \pi (4)^2$$

$$\theta = 107.43^\circ$$

$$15 = \frac{\theta}{2\pi} \pi (4)^2$$

$$\theta = 1.87 \text{ radians}$$

What angle is less than 2π radians and coterminal with $\frac{7}{6}\pi$?

$$-\frac{5\pi}{6} \text{ radians}$$

NO Calculator. Give the exact value of each expression in radians or interval $[0, 2\pi]$.

$$3 \tan^2(x) = 1$$

$$\tan^2 x = \frac{1}{3}$$

$$\tan x = \pm \frac{1}{\sqrt{3}}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\log_{\frac{1}{2}}(\sec x) = -1$$

$$\left(\frac{1}{2}\right)^{-1} = \sec x$$

$$2 = \frac{1}{\cos x}$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

OK Calculator. Give the exact value of each expression over the interval $[0^\circ, 360^\circ]$. Round to nearest tenth.

$$\frac{\frac{1}{2} \csc \theta}{-2} = 6$$

$$\frac{1}{2} \csc \theta = -12$$

$$\csc \theta = -24$$

$$\sin \theta = -\frac{1}{24}$$

$$\theta = -2.39^\circ \rightarrow 50$$

$$180 + 2.39 = 182.39^\circ \text{ or}$$

$$360 - 2.39 = 357.61^\circ$$

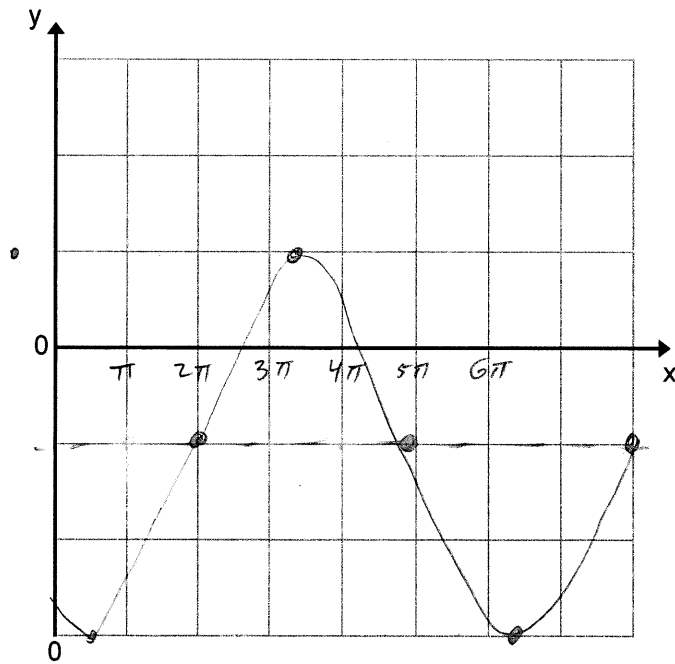
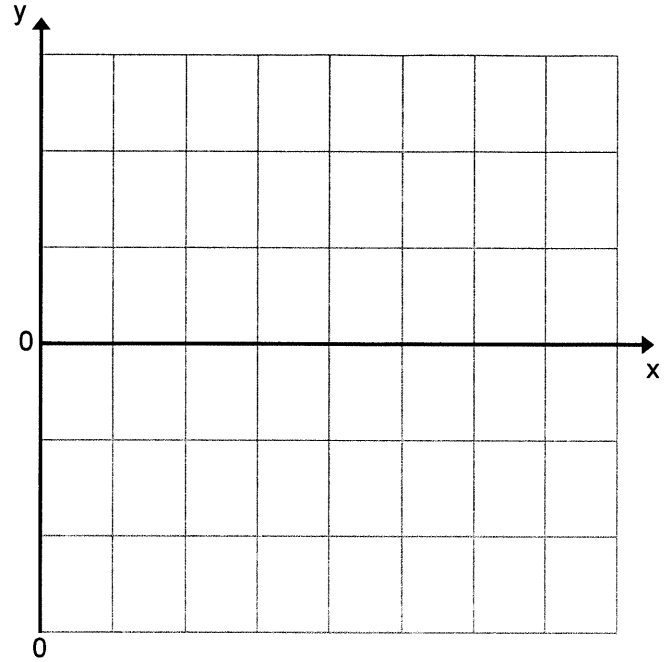
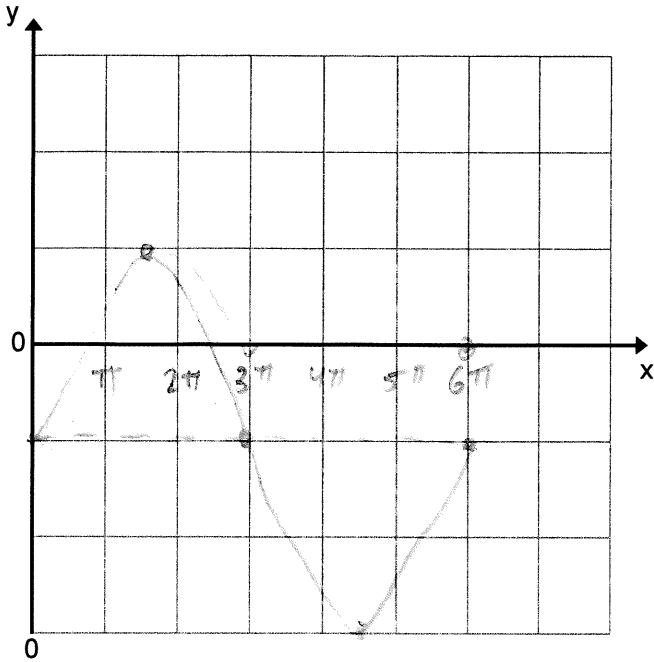
Graph the following with a domain of $[0, 2\pi]$ OR one full period length beginning at 0 radians.

Graph whichever case has larger value.

$$y = 2 \sin\left(\frac{1}{3}x - \frac{2\pi}{3}\right) - 1 = 2 \sin\left(\frac{1}{3}(x - 2\pi)\right) - 1$$

$$\text{Per} = \frac{2\pi}{\frac{1}{3}} = 6\pi$$

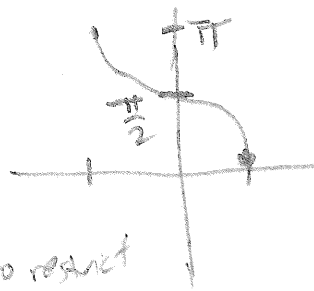
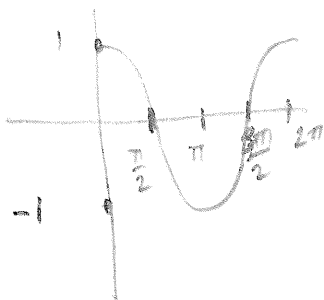
Right 2π



Final

Sketch the graph of $\cos^{-1} x$. Explain why it is necessary to restrict the domain on this graph. What implications does this have when using \cos^{-1} to determine angle measure?

cos x



- only gives an angle between $[0, \pi]$
↳ but really an infinite # of angles

- Need to restrict domain from $[-1, 1]$ so it's a function

OK Calculator. Determine the acute angle, to the nearest tenth, formed by the intersection of the graphs of $5x + 6y = 2$ and $x - 3y = -4$.

$$6y = -5x + 2$$

$$y = \left(-\frac{5}{6}x\right) + \frac{1}{3}$$

$$-3y = -x - 4$$

$$y = \left(\frac{1}{3}x\right) + \frac{4}{3}$$

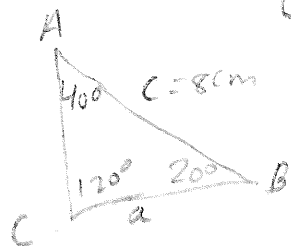
$$\tan^{-1}\left(\frac{-5}{6}\right) = -39.8^\circ$$

$$\tan^{-1}\left(\frac{1}{3}\right) = 18.4^\circ$$

58.2°

"Solve" each triangle with the following conditions if possible. Round angles to the nearest tenth degree and sides to the nearest hundredth.

1. $\angle A = 40^\circ, \angle B = 20^\circ, c = 8\text{cm}$



$$C = 180 - (40 + 20)$$

$$C = 180 - 60$$

$$C = 120^\circ$$

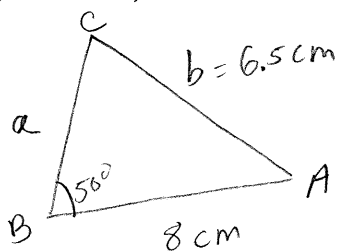
$$\frac{\sin 120}{8} = \frac{\sin 40}{a}$$

$$a = 5.94\text{cm}$$

$$\frac{\sin 120}{8} = \frac{\sin 20}{b}$$

$$b = 3.16\text{cm}$$

2. $\angle B = 50^\circ, b = 6.5\text{cm}, c = 8\text{cm}$



$$\frac{\sin 50}{6.5} = \frac{\sin C}{8}$$

$$C = 70.5^\circ$$

so, $A = 180 - (70.5 + 50)$

$$A = 59.5^\circ$$

$$\frac{\sin 59.5}{a} = \frac{\sin 50}{6.5}$$

$$a = 7.31\text{cm}$$

$$C = 180 - 70.5 = 109.5^\circ$$

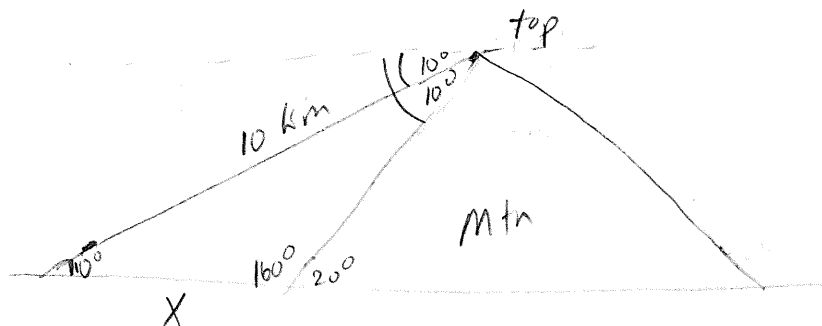
$$A = 180 - (109.5 + 50) = 20.5^\circ$$

$$\frac{\sin 20.5}{a} = \frac{\sin 50}{6.5}$$

$$a = 2.98\text{cm}$$

or

Line of sight to your friend from the top of a mountain is 10km with an angle of depression of 10° . If the mountain maintains a general 20° slope along its side, how far away is your friend from the base of the mountain?



$$\frac{\sin 160}{10} = \frac{\sin 10}{X}$$

$$X = 5.08 \text{ km}$$

COMP 2

NO Calculator. Write an equation of a line with angle of inclination of 15° and contains point $(-2, 3)$.

$\tan X = \text{slope}$

$$\tan 15^\circ = \tan(45^\circ - 30^\circ) = \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1(\frac{1}{\sqrt{3}})} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$Y-3 = \frac{\sqrt{3}-1}{\sqrt{3}+1}(X+2) \iff Y-3 = \frac{3-2\sqrt{3}+1}{2}(X+2) \iff Y-3 = \frac{(2-\sqrt{3})(X+2)}{2}$$

NO Calculator. Determine $\cos 250^\circ \cos 40^\circ + \sin 250^\circ \sin 40^\circ = \cos(250-40) = \cos(210)$

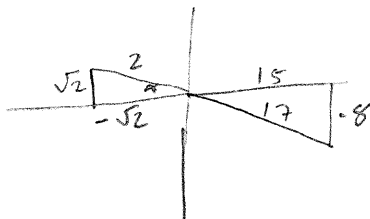
$$= -\cos(30^\circ) = \frac{-\sqrt{3}}{2}$$

Suppose $\tan \alpha = 5$ and $\tan \beta = -\frac{5}{4}$,

Determine $\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} = \frac{5 - (-\frac{5}{4})}{1 + 5(-\frac{5}{4})} = \frac{5 + \frac{5}{4}}{1 - \frac{25}{4}} = \frac{\frac{25}{4}}{-\frac{21}{4}} = \frac{25}{-21}$

NO Calculator. Suppose $\sin \alpha = \frac{\sqrt{2}}{2}$ and $\cos \beta = \frac{15}{17}$, where $\frac{\pi}{2} < \alpha < \pi$ and $\frac{3\pi}{2} < \beta < 2\pi$.

Determine $\sin(\alpha - \beta)$ and $\cos(\alpha + \beta)$



$$\begin{aligned} \sin(\alpha - \beta) &= \sin \alpha \cos \beta - \cos \alpha \sin \beta \\ &= \frac{\sqrt{2}}{2} \left(\frac{15}{17} \right) - \left(-\frac{\sqrt{2}}{2} \right) \left(-\frac{8}{17} \right) \\ &= \frac{15\sqrt{2}}{34} - \frac{8\sqrt{2}}{34} \\ &= \frac{7\sqrt{2}}{34} \end{aligned}$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$= \left(-\frac{\sqrt{2}}{2} \right) \left(\frac{15}{17} \right) - \left(\frac{\sqrt{2}}{2} \right) \left(-\frac{8}{17} \right) = -\frac{15\sqrt{2}}{34} + \frac{8\sqrt{2}}{34} = \frac{-7\sqrt{2}}{34}$$

Write each identity in simplest form. Show your steps.

1. $(\sec x - 1)(\sec x + 1)$

$$\sec^2 x - 1$$

$$(\tan^2 x + 1) - 1$$

$$\boxed{\tan^2 x}$$

2. $1 + \tan^2(90^\circ - x)$

$$1 + \cot^2 x$$

$$\boxed{\csc^2 x}$$

Shown the following trig identities are true.

1. $\csc x(\cos^3 x \tan x - \sin x) = -\sin^2 x$

$$\csc x \left(\cos^3 x \left(\frac{\sin x}{\cos x} \right) - \sin x \right) = -\sin^2 x$$

$$\frac{1}{\sin x} \left(\cos^2 x \sin x - \sin x \right) = -\sin^2 x$$

$$\cos^2 x - 1 = -\sin^2 x \checkmark$$

$$\cos^2 x + \sin^2 x = 1$$

$$\cos^2 x - 1 = -\sin^2 x$$

2. $\frac{1}{1-\sec A} + \frac{1}{1+\sec A} = -2 \cot^2 A$

$$\frac{1+\sec A + 1-\sec A}{1-\sec^2 A} = -2 \cot^2 A$$

$$\frac{2}{1-\sec^2 A} = -2 \cot^2 A$$

$$\frac{2}{1-(1+\tan^2 A)} = -2 \cot^2 A$$

$$\frac{2}{1-1-\tan^2 A} = -2 \cot^2 A$$

$$\frac{2}{-\tan^2 A} = -2 \cot^2 A$$

$$-2 \cot^2 A = -2 \cot^2 A \checkmark$$

OK Calculator. Solve for x over the interval $[0, 2\pi)$. Give answers to the nearest hundredth when necessary.

$2 \cos^2 x + \sin x = -1$

$2(1 - \sin^2 x) + \sin x = -1$

$2 - 2 \sin^2 x + \sin x = -1$

$0 = 2 \sin^2 x - \sin x - 3$

$0 = 2u^2 - u - 3$

$0 = (2u^2 + 2u)(-3u - 3)$

$0 = 2u(u+1) - 3(u+1)$

~~$\begin{matrix} -6 & & 2 \\ -3 & \times & 1 \\ & & -1 \end{matrix}$~~

$0 = (2u-3)(u+1)$

$2 \sin x - 3 = 0$ $\sin x + 1 = 0$

$\sin x = \frac{3}{2}$
 $x = \text{not possible}$

$\sin x = -1$
 $x = \frac{3\pi}{2}$

Solve for x over the interval $[0, 2\pi)$. Give answers to the nearest hundredth when necessary.

$\sin^2 x + 3 \sin x - 1 = 0$

$u = \sin x$

$u^2 + 3u - 1 = 0$

$u = \frac{-3 \pm \sqrt{3^2 - 4(1)(-1)}}{2(1)}$

$u = \frac{-3 \pm \sqrt{9+4}}{2}$

$u = \frac{-3 \pm \sqrt{13}}{2}$

$u = \frac{-3 \pm 3.61}{2}$

$u = .305$

$\sin x = .305$

$x = 1.31$ radians and $x = \pi - 1.31$
 $x = 2.83$ radians

$u = -3.305$
 Not possible

Simplify the following expressions.

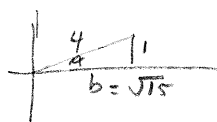
1. $\cos^2 2\theta - \sin^2 2\theta$

$\cos(2(2\theta))$

$\cos(4\theta)$

2. $2 \cos^2 \left(\frac{B}{2}\right) - 1$

$\cos(2\left(\frac{B}{2}\right)) = \cos B$



$$1^2 + b^2 = 4^2$$

$$b^2 = 15$$

$$b = \sqrt{15}$$

NO Calculator. Evaluate both expressions when $\sin \alpha = \frac{1}{4}$ over the interval $[0, \frac{\pi}{2})$.

3. $\frac{\sin 4\alpha}{1 + \cos 4\alpha} = \tan\left(\frac{1}{2}(4\alpha)\right)$

4. $\cos 3\alpha \cos \alpha + \sin 3\alpha \sin \alpha$

$$= \frac{\frac{2}{\sqrt{15}}}{\frac{14}{15}}$$

$$= \frac{2}{\sqrt{15}} \cdot \frac{15}{14}$$

$$= \frac{15}{7\sqrt{15}}$$

$$= \frac{15\sqrt{15}}{7 \cdot 15}$$

$$= \frac{\sqrt{15}}{7}$$

$$= \tan 2\alpha$$

$$= \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

$$= \frac{2 \left(\frac{1}{\sqrt{15}}\right)}{1 - \left(\frac{1}{\sqrt{15}}\right)^2}$$

$$= \frac{\frac{2}{\sqrt{15}}}{1 - \frac{1}{15}}$$

$$= \cos(3\alpha - \alpha)$$

$$= \cos 2\alpha$$

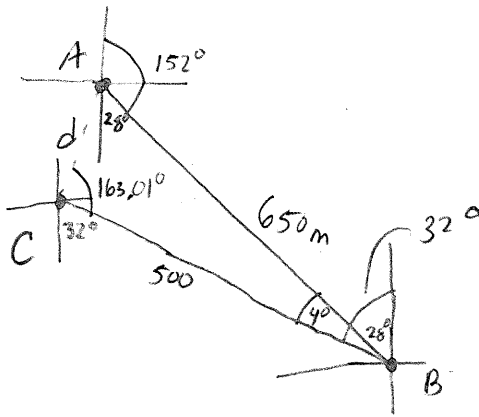
$$= \cos^2 \alpha - \sin^2 \alpha$$

$$= \left(\frac{\sqrt{15}}{4}\right)^2 - \left(\frac{1}{4}\right)^2$$

$$= \frac{15}{16} - \frac{1}{16} = \frac{14}{16} = \frac{7}{8}$$

Calculator OK

Suppose you were to start flying a plane from point A at bearing 152° for 650 miles to reach point B. After reaching point B you turn bearing $N32^\circ W$ and travel for 500 miles to point C. How far and in what direction must you fly to reach point A again? What is the area of $\triangle ABC$?



$$\text{Area} = \frac{1}{2}(500)(650) \sin 40^\circ =$$

$$11,335.43 \text{ mi}^2$$

$$d^2 = 500^2 + 650^2 - 2(500)(650) \cos 40^\circ$$

$$d = 155.19 \text{ miles}$$

$$650^2 = 500^2 + 155.19^2 - 2(155.19)(500) \cos C$$

$$C = 163.01^\circ$$

$$32 + 163.01 = 195.01$$

$$195.01 - 180 = 15.01^\circ \rightarrow$$

Bearing 345° or $N15^\circ W$