Name ______________________________ Date ______________

PreCalc H - 2 – Final - Review

COMP 1

Senior

OK Calculator.

Convert 52.23567° into degrees minutes seconds. *Round to the nearest second.*

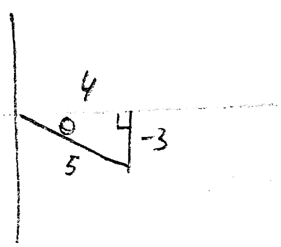
52°, 14 min 8 seconds

Convert $234^\circ 52' 10''$ into decimal degrees. *Round to the nearest hundredth degree.*

$$234^\circ + \frac{52}{60}^\circ + \frac{10}{3600}^\circ = 234.87^\circ$$

No Calculator.

Given that $\sin\theta = -\frac{3}{5}$ and $\frac{3\pi}{2} \le \theta \le 2\pi$. Determine all five other trigonometric relations.



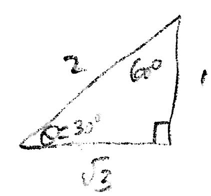
$\sin\theta = -\frac{3}{5}$ $\quad \csc\theta = -\frac{5}{3}$

$\cos\theta = \frac{4}{5}$ $\quad \sec\theta = \frac{5}{4}$

$\tan\theta = \frac{-3}{4}$ $\quad \cot\theta = -\frac{4}{3}$

Determine all values of $Cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \theta$

$Cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \theta \rightarrow$ $\quad \cos\theta = -\frac{\sqrt{3}}{2}$



$\theta = 150^\circ \pm 360n$

$\theta = 210^\circ \pm 360n$

or

$\theta = \frac{5\pi}{6} \pm 2\pi n$

$\theta = \frac{7\pi}{6} \pm 2\pi n$

$\{n \mid n \in \mathbb{Z}\}$

OK Calculator.

Fill in the blank with best answer.

The ___Unit circle___ has a radius of 1 unit and is centered at the origin.

The ___$-\cot\theta$___ measures the slope of a tangent line to the unit circle.

The ___$\sin\theta$___ determines the y-coordinate of a point on the unit circle.

The ___$\cos\theta$___ determines the x-coordinate of a point on the unit circle.

Re-write each angle measure using the other unit, either degrees or radians. SHOW WORK.

1. 200^o

$\frac{200}{180} = \frac{X}{\pi}$

$X = 3.49$ radians

2. $\frac{5\pi}{2}$

$\frac{X}{180°} = \frac{\frac{5\pi}{2}}{\pi}$

$X = 450°$

3. 4

$\frac{X}{180} = \frac{4}{\pi}$

$X = 229.18°$

A sector of a circle has area $15cm^2$ and radius 4 cm. Find central angle to the nearest tenth unit.

$15 = \frac{\theta}{360}\pi(4)^2$

$107.43° = \theta$

$15 = \frac{\theta}{2\pi}\pi(4)^2$

$\theta = 1.87$ radians

What angle is less than 2π radians and coterminal with $\frac{7}{6}\pi$?

$-\frac{5\pi}{6}$ radians

NO Calculator. Give the exact value of each expression in radians or interval $[0, 2\pi]$.

$3\tan^2(x) = 1$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm\frac{1}{\sqrt{3}}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$\log_{\frac{1}{2}}(\sec x) = -1$

$\left(\frac{1}{2}\right)^{-1} = \sec x$

$2 = \frac{1}{\cos x}$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

OK Calculator. Give the exact value of each expression over the interval $[0°, 360°]$. Round to nearest tenth.

$\frac{\frac{1}{2}\csc\theta}{-2} = 6$

$\frac{1}{2}\csc\theta = -12$

$\csc\theta = -24$

$\sin\theta = -\frac{1}{24}$

$\theta = -2.39°$ so

$180 + 2.39 = 182.39°$ or

$360 - 2.39 = 357.61°$

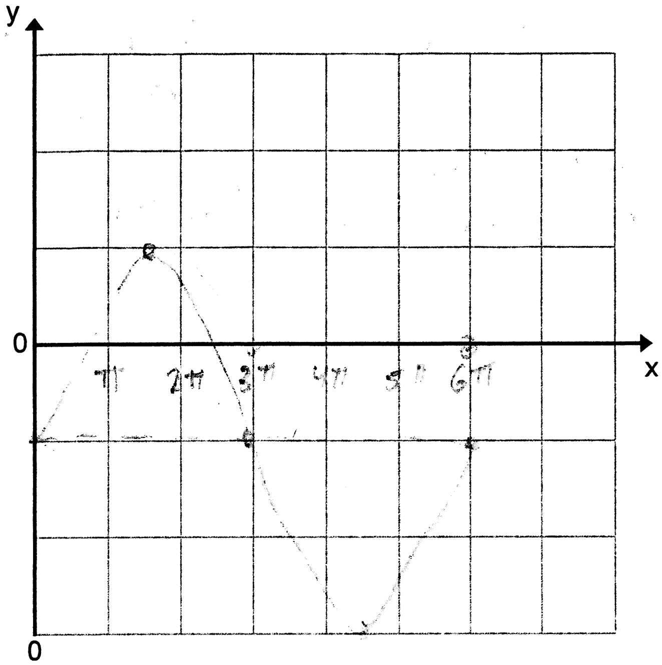
Graph the following with a domain of $[0,2\pi]$ OR one full period length beginning at 0 radians.

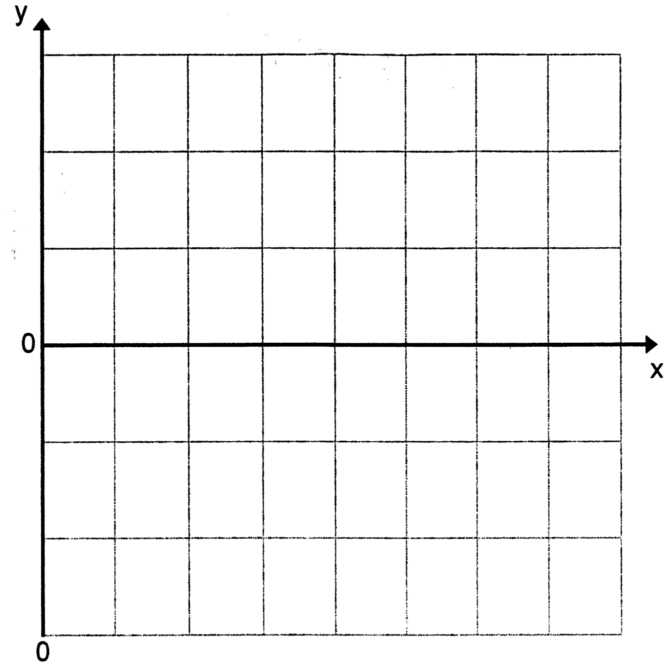
<u>Graph whichever case has larger value.</u>

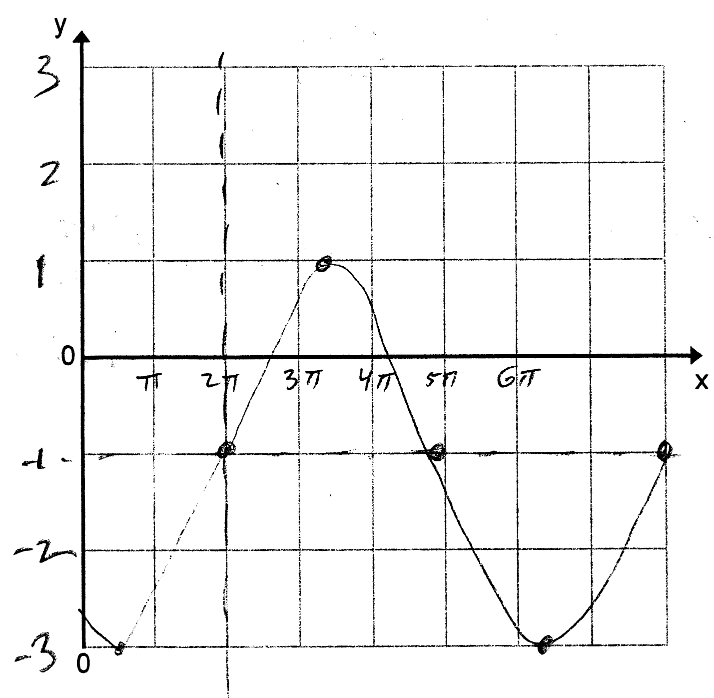
$y=2\sin\left(\frac{1}{3}x-\frac{2\pi}{3}\right)-1 \quad = 2\sin\left(\frac{1}{3}(x-2\pi)\right)-1$

$\text{Per} = \frac{2\pi}{1/3} = 6\pi$

Right 2π



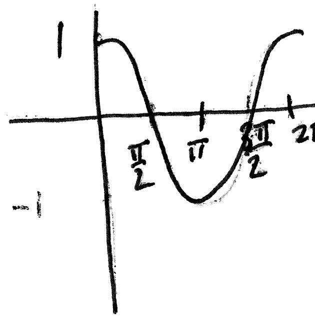


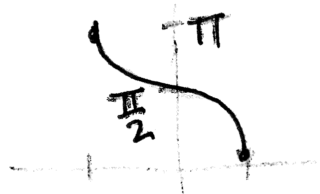


Final

Sketch the graph of $\cos^{-1} x$. Explain why it is necessary to restrict the ~~domain~~ range on this graph. What implications does this have when using $\cos^{-1}$ to determine angle measure?

Cos X





-only gives an angle between $[0, \pi]$

↳ but really an infinite # of angles

-Need to restrict ~~domain~~ range from $[0, \pi]$ so it's a function

OK Calculator. Determine the acute angle, to the nearest tenth, formed by the intersection of the graphs of $5x + 6y = 2$ and $x - 3y = -4$.

$6y = -5x + 2$ $\quad$ $y = \left(-\frac{5}{6}x\right) + \frac{1}{3}$

$-3y = -x - 4$ $\quad$ $y = \left(\frac{1}{3}x\right) + \frac{4}{3}$

$\tan^{-1}\left(-\frac{5}{6}\right) = -39.8°$

$\tan^{-1}\left(\frac{1}{3}\right) = 18.4$ $\quad$ $> 58.2°$

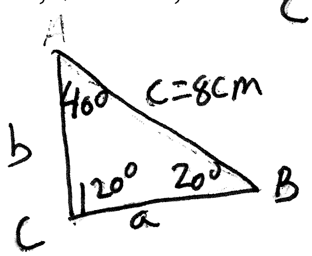
"Solve" each triangle with the following conditions if possible. Round angles to the nearest tenth degree and sides to the nearest hundredth.

1. $\measuredangle A = 40°, \measuredangle B = 20°, c = 8cm$

$C = 180 - (40 + 20)$

$C = 180 - 60$

$C = 120°$



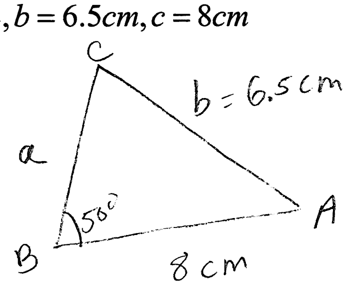
$\frac{\sin 120}{8} = \frac{\sin 40}{a}$

$a = 5.94$ cm

$\frac{\sin 120}{8} = \frac{\sin 20}{b}$

$b = 3.16$cm

2. $\measuredangle B = 50°, b = 6.5cm, c = 8cm$



$\frac{\sin 50}{6.5} = \frac{\sin C}{8}$

$\boxed{C = 70.5°}$

so, $A = 180 - (70.53 + 50)$

$\boxed{A = 59.5°}$

$\frac{\sin 59.47}{a} = \frac{\sin 50}{6.5}$

$\boxed{a = 7.31 \text{ cm}}$

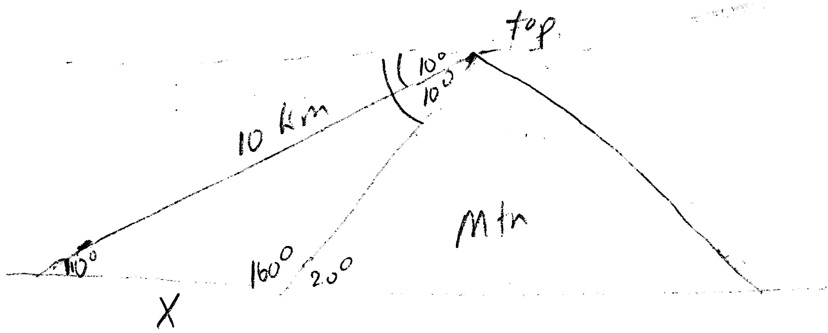
or

$C = 180 - 70.53 = 109.5°$

$A = 180 - (109.47 + 50) = 20.5°$

$\frac{\sin 20.5}{a} = \frac{\sin 50}{6.5}$ $\quad$ $a = 2.98$ cm

Line of sight to your friend from the top of a mountain is 10km with an angle of depression of 10°. If the mountain maintains a general 20° slope along its side, how far away is your friend from the base of the mountain?



$$\frac{\sin 160}{10} = \frac{\sin 10}{X}$$

$X = 5.08\text{ km}$

COMP 2

NO Calculator. Write an equation of a line with angle of inclination of 15° and contains point $(-2,3)$.

$\tan x = \text{slope}$

$$\tan 15^\circ = \tan(45^\circ - 30^\circ) = \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1\left(\frac{1}{\sqrt{3}}\right)} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$y - 3 = \frac{\sqrt{3}-1}{\sqrt{3}+1}(x+2) \leftrightarrow y - 3 = \frac{3 - 2\sqrt{3} + 1}{2}(x+2) \leftrightarrow \boxed{y - 3 = (2 - \sqrt{3})(x+2)}$$

NO Calculator. Determine $\cos 250^\circ \cos 40^\circ + \sin 250^\circ \sin 40^\circ$

$$= \cos(250 - 40) = \cos(210)$$

$$= -\cos(30)^\circ = -\frac{\sqrt{3}}{2}$$

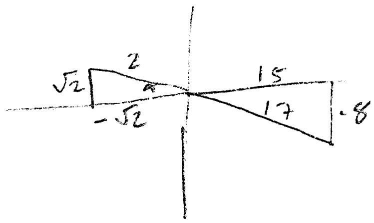
Suppose $\tan\alpha = 5$ **and** $\tan\beta = -\frac{5}{4}$,

Determine $\tan(\alpha-\beta)$ $= \dfrac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta} = \dfrac{5-\left(-\frac{5}{4}\right)}{1+5\left(-\frac{5}{4}\right)} = \dfrac{\frac{25}{4}}{-\frac{21}{4}}$

$= -\dfrac{25}{21}$

NO Calculator. Suppose $\sin\alpha = \frac{\sqrt{2}}{2}$ and $\cos\beta = \frac{15}{17}$, where $\frac{\pi}{2} < \alpha < \pi$ and $\frac{3\pi}{2} < \beta < 2\pi$.

Determine $\sin(\alpha-\beta)$ and $\cos(\alpha+\beta)$



$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \frac{\sqrt{2}}{2}\left(\frac{15}{17}\right) - \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{-8}{17}\right)$

$= \dfrac{15\sqrt{2}}{34} - \dfrac{8\sqrt{2}}{34}$

$= \boxed{\dfrac{7\sqrt{2}}{34}}$

$\cos(\alpha+\beta)$: $\cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{15}{17}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{8}{17}\right) = \dfrac{-15\sqrt{2}}{34} + \dfrac{8\sqrt{2}}{34} = \boxed{\dfrac{-7\sqrt{2}}{34}}$

Write each identity in simplest form. Show you steps.

1. $(\sec x - 1)(\sec x + 1)$

$\sec^2 x - 1$

$(\tan^2 x + 1) - 1$

$\boxed{\tan^2 x}$

2. $1 + \tan^2(90^\circ - x)$

$1 + \cot^2 x$

$\boxed{\csc^2 x}$

$\cos^2 x + \sin^2 x = 1$

$1+\tan^2 x = \frac{1}{\cos^2 x}$

$1+\tan^2 x = \sec^2 x$

Shown the following trig identities are true.

1. $\csc x(\cos^3 x \tan x - \sin x) = -\sin^2 x$

$\csc x\left(\cos^3 x\left(\frac{\sin x}{\cos x}\right) - \sin x\right) = -\sin^2 x$

$\frac{1}{\sin x}\left(\cos^2 x \sin x - \sin x\right) = -\sin^2 x$

$\cos^2 x - 1 = -\sin^2 x$ ✓

$\cos^2 x + \sin^2 x = 1$

$\cos^2 x - 1 = -\sin^2 x$

2. $\frac{1}{1-\sec A} + \frac{1}{1+\sec A} = -2\cot^2 A$

$\frac{1+\sec A + 1 - \sec A}{1-\sec^2 A} = -2\cot^2 A$

$\frac{2}{1-\sec^2 A} = -2\cot^2 A$

$\frac{2}{1-(1+\tan^2 A)} = -2\cot^2 A$

$\frac{2}{1-1-\tan^2 A} = -2\cot^2 A$

$\frac{2}{-\tan^2 A} = -2\cot^2 A$

$-2\cot^2 A = -2\cot^2 A$ ✓

OK Calculator. Solve for x over the interval $[0, 2\pi)$. Give answers to the nearest hundredth when necessary.

$2\cos^2 x + \sin x = -1$

$2(1-\sin^2 x) + \sin x = -1$

$2 - 2\sin^2 + \sin x = -1$

$0 = 2\sin^2 x - \sin x - 3$

$u = \sin x$

$0 = 2u^2 - u - 3$

$0 = (2u^2 + 2u)(-3u - 3)$

$0 = 2u(u+1) - 3(u+1)$

-6, -3, 2, -1

$0 = (2u-3)(u+1)$

$2\sin x - 3 = 0$

$\sin x = \frac{3}{2}$

$x =$ Not possible

$\sin x + 1 = 0$

$\sin x = -1$

$x = \frac{3\pi}{2}$

Solve for x over the interval $[0, 2\pi)$. Give answers to the nearest hundredth when necessary.

$\sin^2 x + 3\sin x - 1 = 0$ $\quad u = \sin x$

$u^2 + 3u - 1 = 0$

$u = \frac{-3 \pm \sqrt{3^2 - 4(1)(-1)}}{2(1)}$

$u = \frac{-3 \pm \sqrt{9+4}}{2}$

$u = \frac{-3 \pm \sqrt{13}}{2}$

$u = \frac{-3 \pm 3.61}{2}$

$u = .305$

$u = -3.305$ not possible

$\sin x = .305$

$x = .31$ radians and $x = \pi - .31$

$x = 2.83$ radians

Simplify the following expressions.

1. $\cos^2 2\theta - \sin^2 2\theta$

$\cos(2(2\theta))$

$\cos(4\theta)$

2. $2\cos^2\left(\frac{\beta}{2}\right) - 1$

$\cos\left(2\left(\frac{\beta}{2}\right)\right) = \cos\beta$

$\frac{4}{a}$ 11 $b = \sqrt{15}$

$1^2 + b^2 = 4^2$
$b^2 = 15$
$b = \sqrt{15}$

NO Calculator. **Evaluate both expressions when** $\sin\alpha = \frac{1}{4}$ **over the interval** $[0, \frac{\pi}{2})$.

3. $\frac{\sin 4\alpha}{1+\cos 4\alpha} = \tan\left(\frac{1}{2}(4x)\right)$

$= \tan 2x$

$= \frac{2\tan\alpha}{1-\tan^2\alpha}$

$= \frac{2\left(\frac{1}{\sqrt{15}}\right)}{1-\left(\frac{1}{\sqrt{15}}\right)^2}$

$= \frac{\frac{2}{\sqrt{15}}}{1-\frac{1}{15}}$

$= \frac{\frac{2}{\sqrt{15}}}{\frac{14}{15}}$

$= \frac{2}{\sqrt{15}} \cdot \frac{15}{14}$

$= \frac{15}{7\sqrt{15}}$

$= \frac{15\sqrt{15}}{7 \cdot 15}$

$= \frac{\sqrt{15}}{7}$

4. $\cos 3\alpha \cos\alpha + \sin 3\alpha \sin\alpha$

$= \cos(3\alpha - \alpha)$

$= \cos 2\alpha$

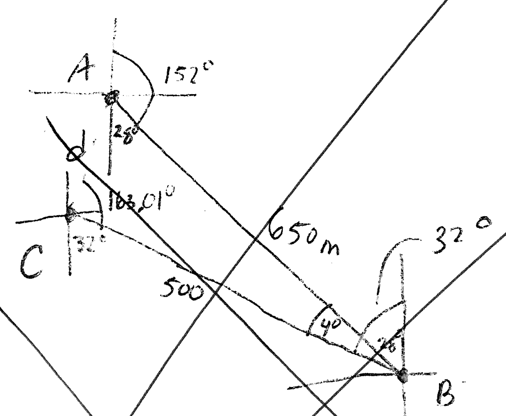
$= \cos^2\alpha - \sin^2\alpha$

$= \left(\frac{\sqrt{15}}{4}\right)^2 - \left(\frac{1}{4}\right)^2$

$= \frac{15}{16} - \frac{1}{16} = \frac{14}{16} = \frac{7}{8}$

Calculator OK

Suppose you were to start flying a plane from point A at bearing 152° **for 650 miles to reach point B. After reaching point B you turn bearing** $N32^\circ W$ **and travel for 500 miles to point C. How far an in what direction must you fly to reach point A again? What is the area of** $\triangle ABC$**?**



Area $= \frac{1}{2}(500)(650)\sin 4 =$

11,335.43 mi²

$d^2 = 500^2 + 650^2 - 2(500)(650)\cos 4$

$d = 155.19$ miles

$650^2 = 500^2 + 155.19^2 - 2(155.19)(500)\cos C$

$C = 163.01^\circ$

$32 + 163.01 = 195.01$

$195.01 - 180 = 15.01^\circ \longrightarrow$ Bearing 345° or N15°W